The Time Domain Characterization of Radio Links

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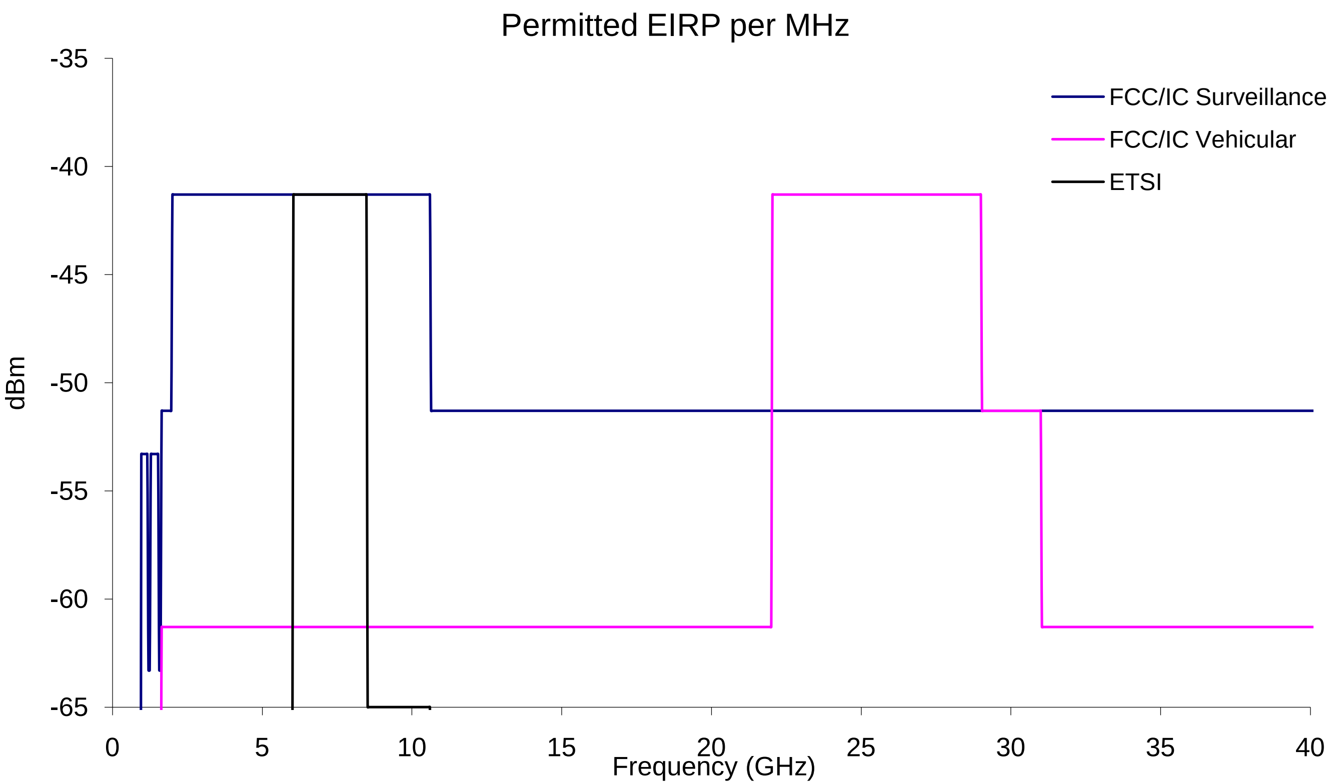
Purpose of Investigating Time Domain Characteristics

- Very wide bandwidth radio links can be used for high data rate communications,
  - Sub-nanosecond pulse widths allow data rates of gigabits per second,
  - Direct baseband communications is possible, using pulses without the need for RF modulation and demodulation.
- Radio pulses can be used for accurate ranging,
  - 3 cm resolution is achievable using 0.1 nsec pulses,
  - 10 GHz bandwidth are needed.
- Physics of radiation and antenna behaviour can be explored,
  - Time domain characterization helps in understanding the radiation process,
  - The use of antennas for handling extremely wide bandwidths and ultra short response times can be explored.

Motivation from a Frequency Domain Perspective

- Large amounts of spectrum have been made available,
  - The FCC and IC in North America, ETSI in Europe, have set limits on the allowed EIRP (in the form of dBm/MHz) in license-exempt applications over very wide bandwidths.
- Antennas are being developed with desirable characteristics over very wide bandwidths,
  - Vivaldi and fractal antennas,
  - The humble thin-wire dipole – no kidding, stay tuned
- Radio link analysis to date has been done almost exclusively in the frequency domain,
  - There is a large body of research available on antennas and their use in radio links over wide bandwidths.
  - Is there a better way to reveal truly wideband behaviour, with a focus on antennas that operate, according to the EIRP limits, in the range of 1 to 10 GHz?

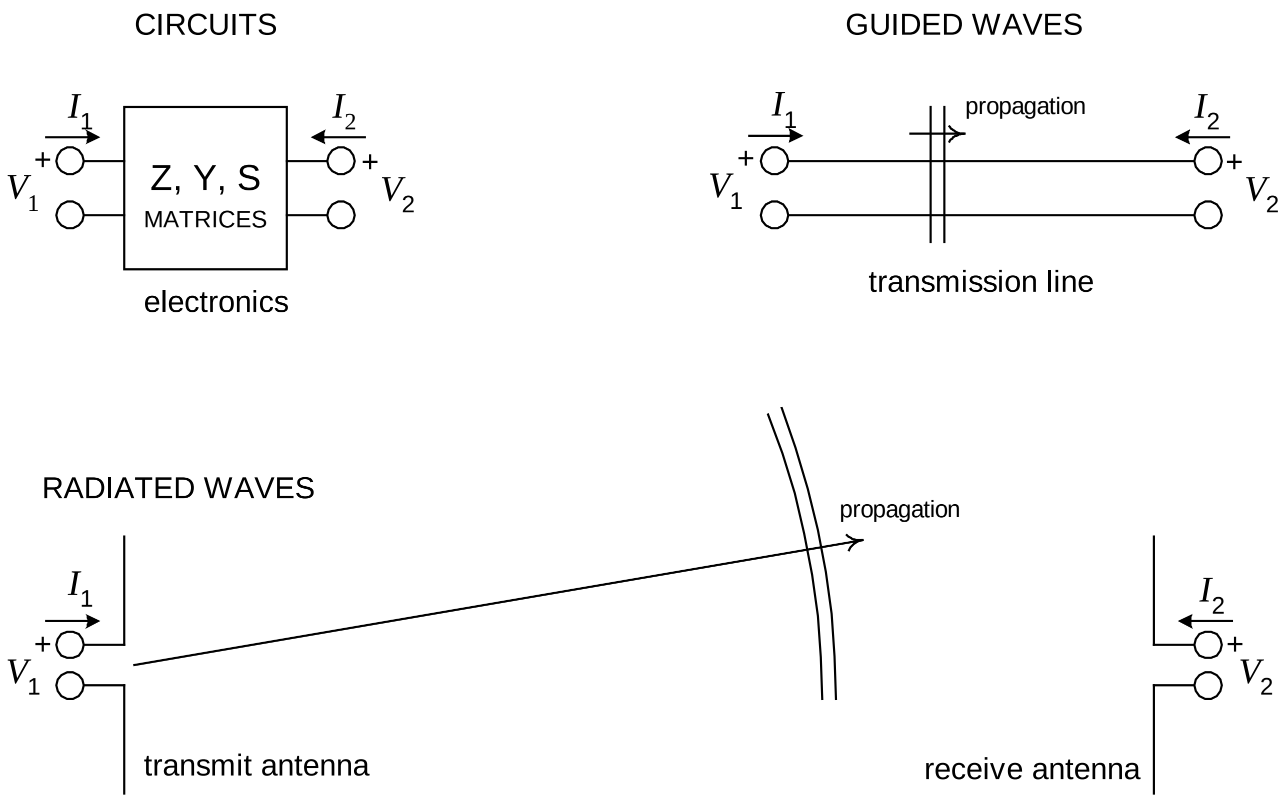
EIRP Limits Set By Regulatory Authorities



Approach to Analyzing Radio Links

- Treat a radio link as a two port network,
  - In the frequency domain use standard definitions of impedance (or admittance), and standard expressions for the electric field (in the far field),
  - Derive and expression for received port current versus transmit port voltage,
    - Well known link formulas (e.g. Friis transmission formula) are readily derived from these expressions.
- Establish an equivalent expression for the time domain,
  - Utilize Fourier transform relationships to define time domain parameters from frequency domain functions,
  - Derive a transfer function for a radio link.

Network Representation of a Radio Link



Frequency Domain Characterization of a Radio Link

Start by assuming that the current density distribution set up by an antenna is known for a given applied source voltage,

Find the electric field radiated by the antenna due to this current density distribution,

Find the voltage and current induced in a receiving antenna,

Express the result in the form of a transfer function (e.g. received current for an applied voltage).

Current Density and Vector Potential

$\mathbf{J}_s$ is the source current density distribution set up in space by a particular antenna.

The vector potential due to this current density is,

$$\mathbf{A} = \frac{\mu}{4\pi} \iiint \frac{\mathbf{J}_s e^{-j\omega r/c}}{r} \, dv$$

Electric Field Radiated by an Antenna

The electric field is found from,

$$\mathbf{E} = -j\omega\mathbf{A}$$

$$= \frac{-j\omega\mu}{4\pi}\iiint\frac{\mathbf{J}_s e^{-j\omega r/c}}{r}\,dv$$

$$= -\frac{10^{-7}}{r}(j\omega)\iiint\mathbf{J}_s e^{-j\omega r/c}\,dv$$

for r large

Characterizing the Antenna

Define an ***H***-function for an antenna

$$\boldsymbol{H} = j\omega\left[\frac{\iiint \mathbf{J}_s\, \mathrm{e}^{-j\omega \mathrm{r/c}} dv}{I_s}\right] = j\omega\, \mathbf{l}_{eff}$$

Captures the effects of the spatial current distribution at a given frequency,

Applies to an antenna as a transmitter or a receiver

Radiated Electric Field

Electric field at the transmit port is then

$$\mathbf{E}_t = -\frac{10^{-7}}{r} I_s \boldsymbol{H}_t$$

$$= -\frac{10^{-7}}{r} V_s Y_t \boldsymbol{H}_t$$

V_s the source voltage and

Y_t is the port admittance

Receive Open-Circuit Voltage

The open-circuit voltage at an antenna receive port r metres away (in the far field of both antennas), due to this electric field, is

$$V_{r(oc)} = -\mathbf{E}_t \cdot \mathbf{l}_{eff\ r}$$

$$= \frac{10^{-7}}{r} V_s Y_t \boldsymbol{H}_t \cdot \frac{\boldsymbol{H}_r}{j\omega}$$

Current at Receive Port

The current at a receive port r meters away (in the far field of both antennas) is

$$I_r = \frac{10^{-7}}{r}\left(\frac{1}{j\omega}\right) Y_t \, Y_r \, \boldsymbol{H}_t \cdot \boldsymbol{H}_r \, V_s$$

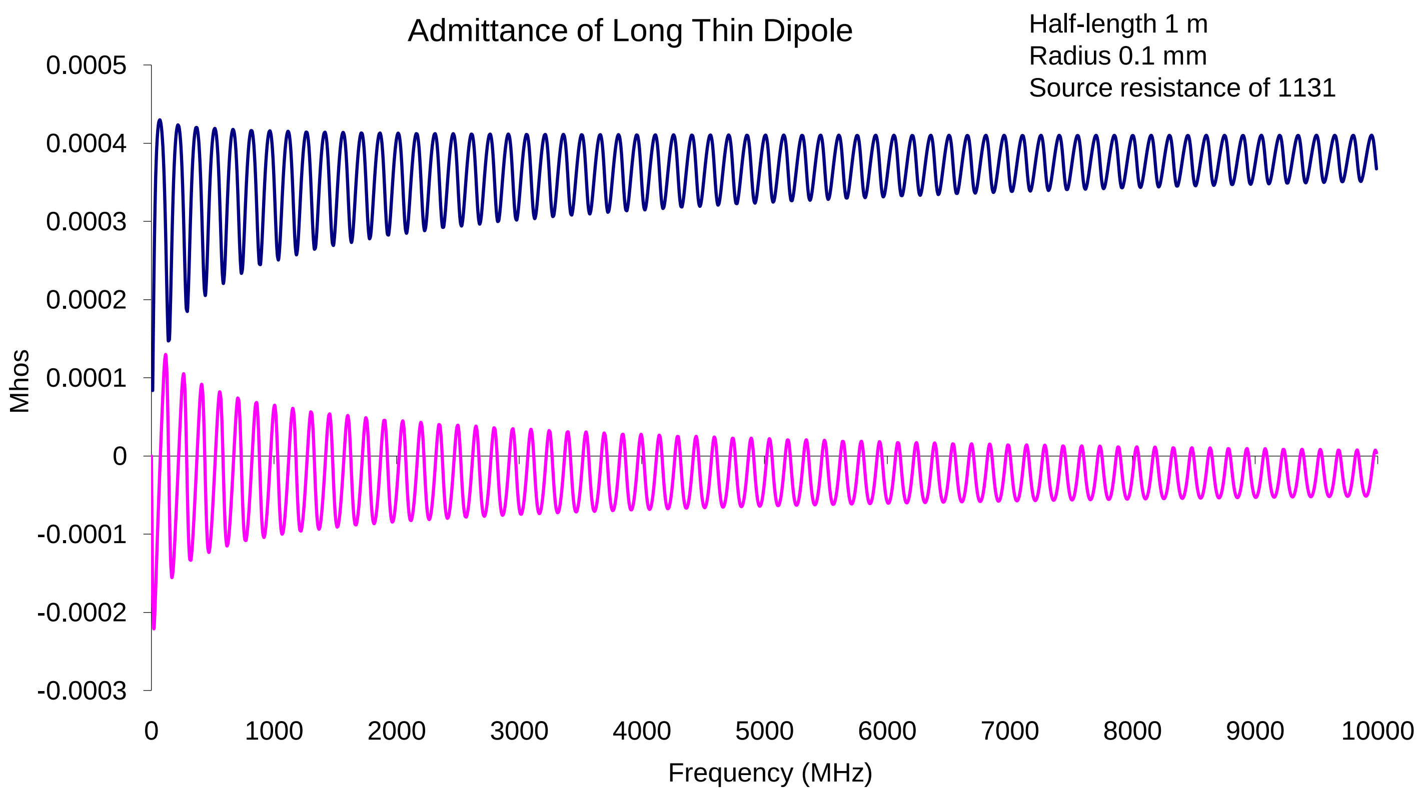
Radio Link As a Two-Port Network

Current at the receive port for a unit voltage applied at the transmit port,

$$Y_{21} = \frac{10^{-7}}{r}\left(\frac{1}{j\omega}\right) Y_t \, Y_r \, \boldsymbol{H}_t \cdot \boldsymbol{H}_r$$

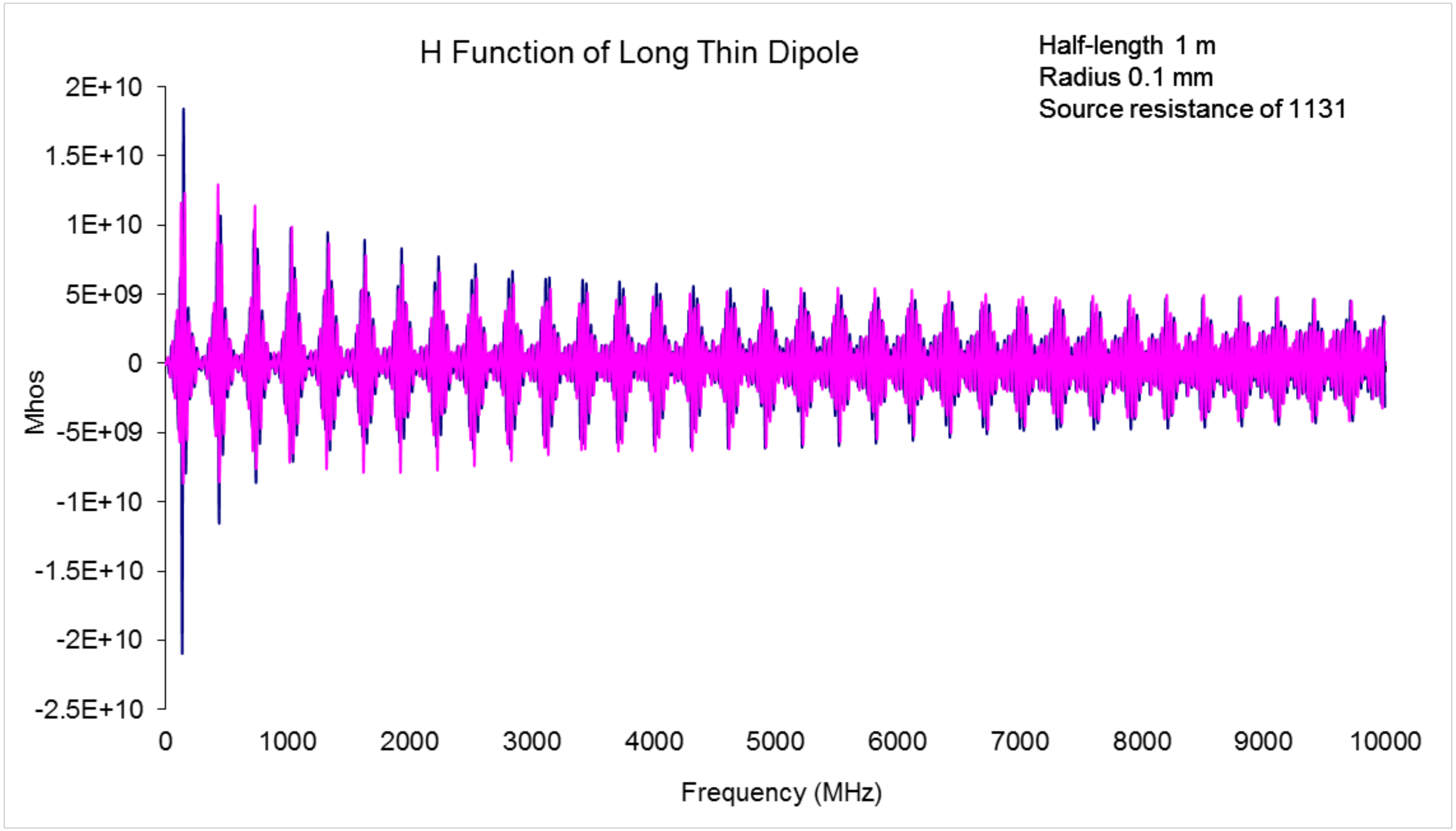
Example of Frequency Domain *Y* Function

(Admittance)



Example of Frequency Domain **H** function

(Electric Field in Far Field for 1 A Source)



Time Domain Characterization of a Radio Link

Start by assuming the charge density distribution set up by an antenna is known for a given applied source voltage,

Find the electric field radiated by the antenna due to this charge density distribution,

Find the voltage and current induced in a receiving antenna,

Express the result in the form of a transfer function (e.g. received current waveform for a given applied voltage waveform).

Source Distribution and Electric Field

q_s is the source charge distribution.

Electric field could be determined from,

$$e = \frac{10^{-7}}{r} \iiint q_s \left\{ \frac{\hat{\mathbf{r}} \times \left[\left(\hat{\mathbf{r}} - \mathbf{v}/c \right) \times \mathbf{a} \right]}{\left(1 - \hat{\mathbf{r}} \cdot \mathbf{v}/c \right)^3} \right\} dv$$

evaluated at $t = t_o - r(t)/c$,
where r is assumed constant over the integration volume,

Characterizing the Antenna

Instead of finding the electric field directly, define two functions as follows,

y function (admissivity)

Inverse Fourier transform of the admittance, *Y*,

h function (directionality)

Electric field ($\times 10^7\, r$) due to a unit current impulse, or

the inverse Fourier transform of the ***H*** function (i.e. time derivative of the inverse Fourier transform of the effective length of the antenna).

Electric Field

Electric field could then be found from the **h** and *y* functions,

$$\mathbf{e} = -\frac{10^{-7}}{r} y * \mathbf{h}$$

but it is not needed directly.

Link Function

A link function, *l*, can be identified as the convolution of the respective transmit and receive functions (concurrent with a dot product):

$$
\begin{aligned}
l &= \left(y_t * \mathbf{h}_t\right) \otimes \left(y_r * \mathbf{h}_r\right) \\
&= y_t * y_r * \mathbf{h}_t \otimes \mathbf{h}_r
\end{aligned}
$$

Received Current

Then the current at the receive port is proportional to the time integral of the convolution of the source voltage and link function:

$$i_r = \frac{10^{-7}}{r}\int l * v_s \; dt = \frac{10^{-7}}{r}\int y_t * y_r * \mathbf{h}_t \otimes \mathbf{h}_r * v_s \; dt$$

(evaluated at the retarded time $t = t_0 - r(t) / c$)

Radio Link as a Two Port Network in the Time Domain

The transfer function, y_{21}, for the link is then the received current for an impulse transmit voltage:

$$y_{21} = \frac{10^{-7}}{r} \int y_t * y_r * \mathbf{h}_t \otimes \mathbf{h}_r \, dt$$

(evaluated at the retarded time $t = t_0 - r(t) / c$)

What is the Form of These Characterizing Functions for Various Antennas ?

Calculate and compare the *y* and **h** functions for:

- Current element,
- Short dipole,
- Long dipole,
  - Real current distribution,
  - Sinusoidal current filament (SCF) distribution,
- Infinite dipole
  - Cylindrical,
  - Conical.

Calculation Methods

Analytical expressions,

- Current element and short dipole,
- Long and infinite dipoles,

Method-of-moments,

- Long dipole, cylindrical and conical,
- Approximation to infinite dipoles.

Current Element

Fundamental source entity,

$$I_s = I_o d\mathbf{s}$$

$$i_s = i_o d\mathbf{s}\,\delta(t)$$

Electric Field of a Current Element

Electric field (90 deg. to element axis),

$$\mathbf{E} = \frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r} I_o d\mathbf{s}$$

$$\mathbf{e} = \frac{10^{-7}}{r} i_o d\mathbf{s}\, \delta'(t - r/c)$$

y and **h** Functions of a Current Element

y function,

$$y = \delta(t) \qquad \text{(by definition)}$$

h function

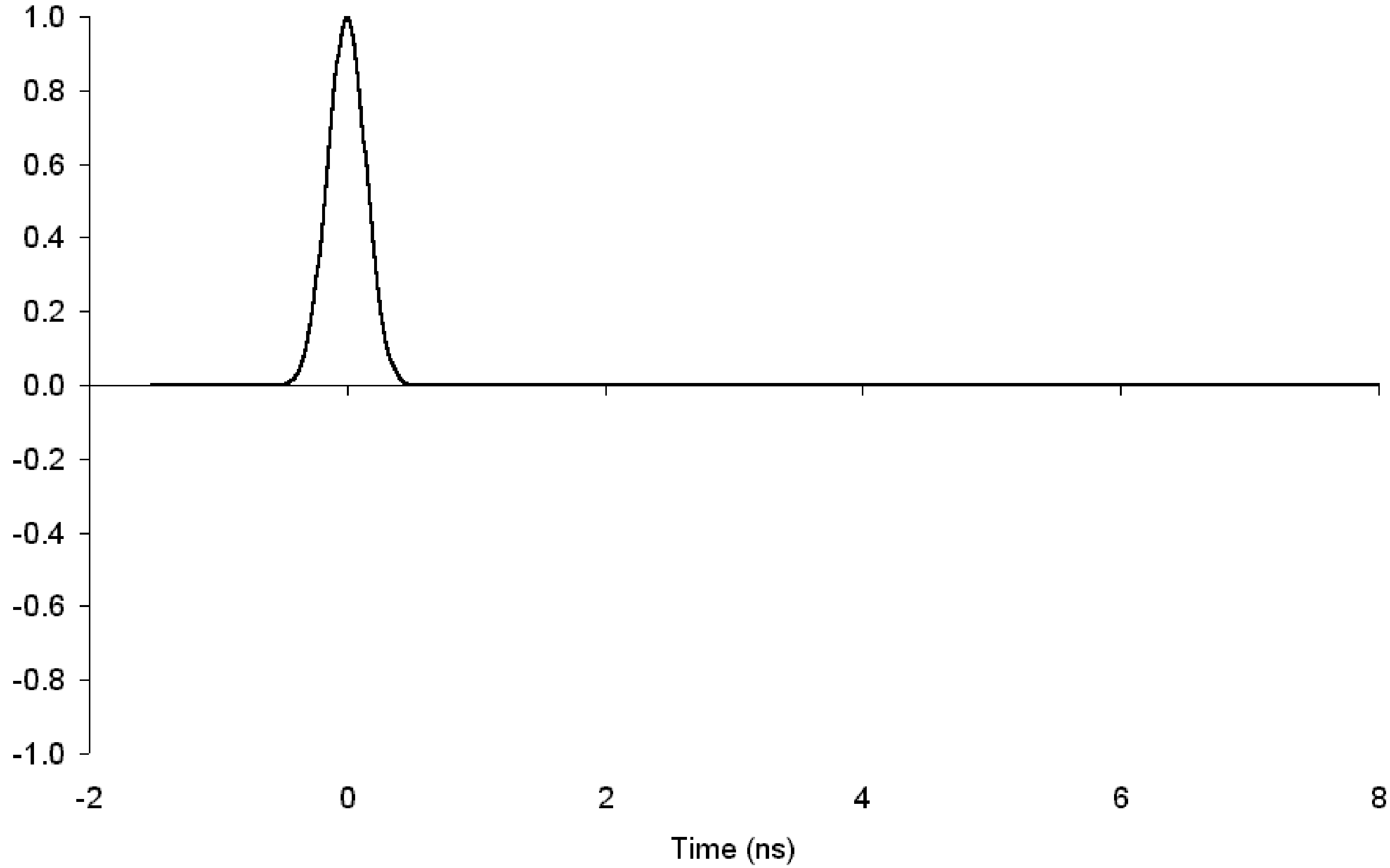
$$\boldsymbol{H} = j\omega\, d\mathbf{s}$$

$$\mathbf{h} = d\mathbf{s}\, \delta'(t - r/c)$$

$$\mathrm{y} * \mathbf{h} = d\mathbf{s}\, \delta'(t - r/c)$$

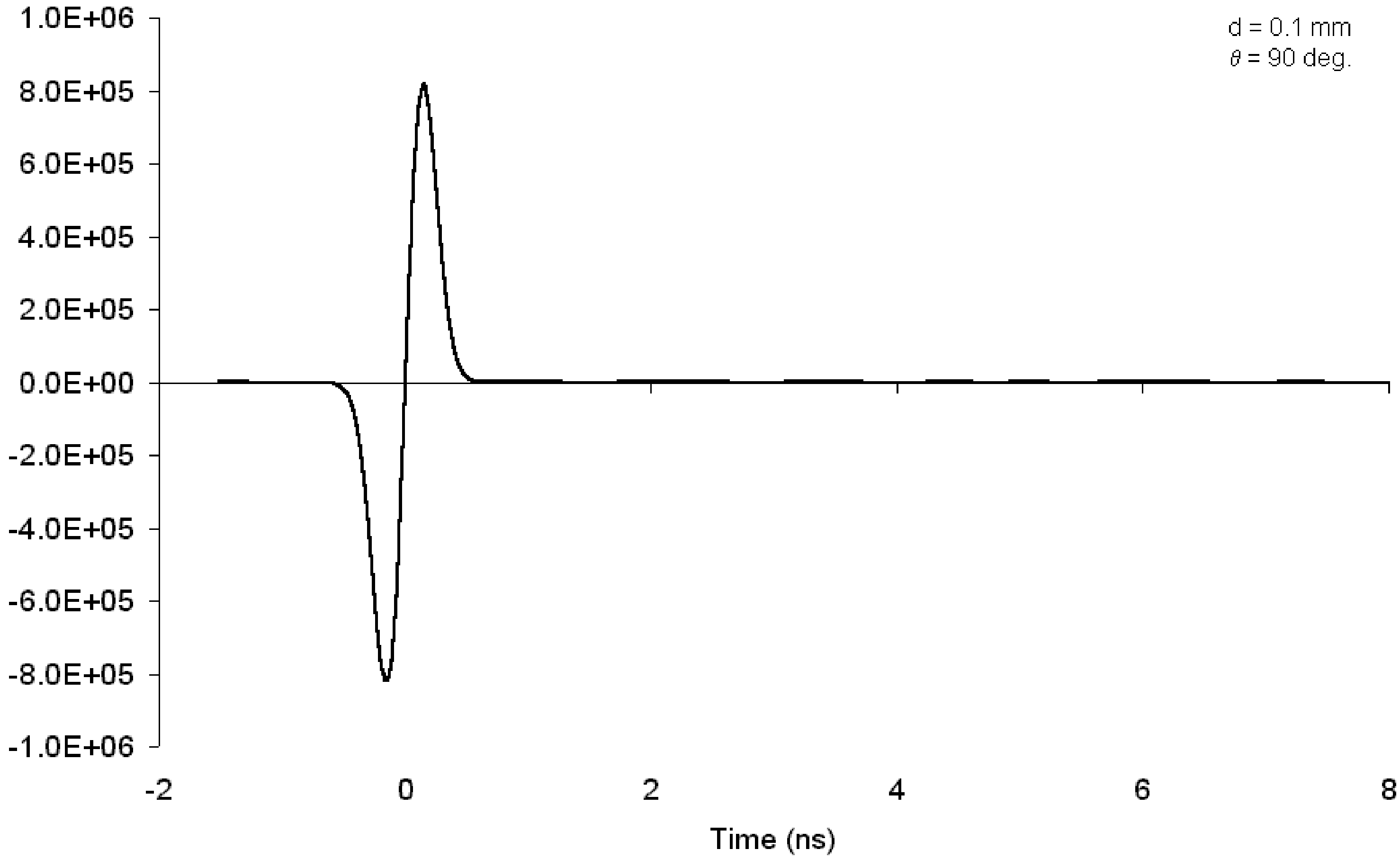
y Function of a Current Element

(with Gaussian source)



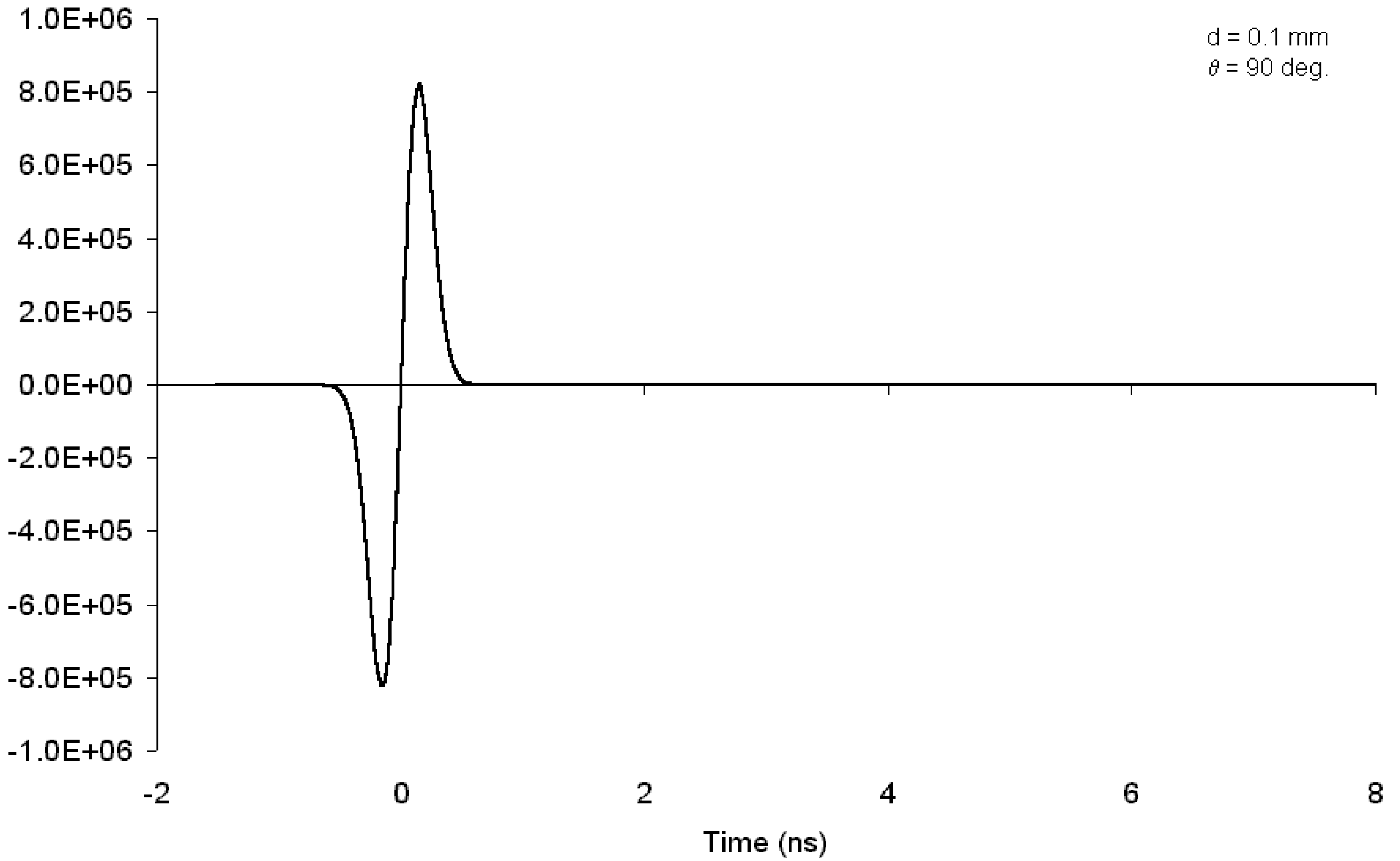
h Function of a Current Element

(with Gaussian source)



$y_* \mathbf{h}$ of a Current Element

(with Gaussian source)



Short Dipole

“Short” means that the wavelength of highest frequency of interest is much longer than the dipole,

“Short” means that travel time along length is much smaller than the shortest time interval of interest,

At all frequencies of interest the current distribution along the dipole is “triangular”,

The impedance is virtually pure reactance at all frequencies of interest,

Example used is length $s = 1.0$ mm, radius $a = 0.1$ mm.

Electric Field of a Short Dipole

Electric field in frequency-domain (90 deg. to wire axis),

$$\mathbf{E} = \frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r} I_o \mathbf{s}$$

$$\mathbf{e} = \frac{10^{-7}}{r} i_o \mathbf{s}\delta'\left(t - r/c\right)$$

y and **h** Functions of a Short Dipole

y function

$$Y = j\omega C_o$$

$$y = C_o \delta'(t)$$

h function,

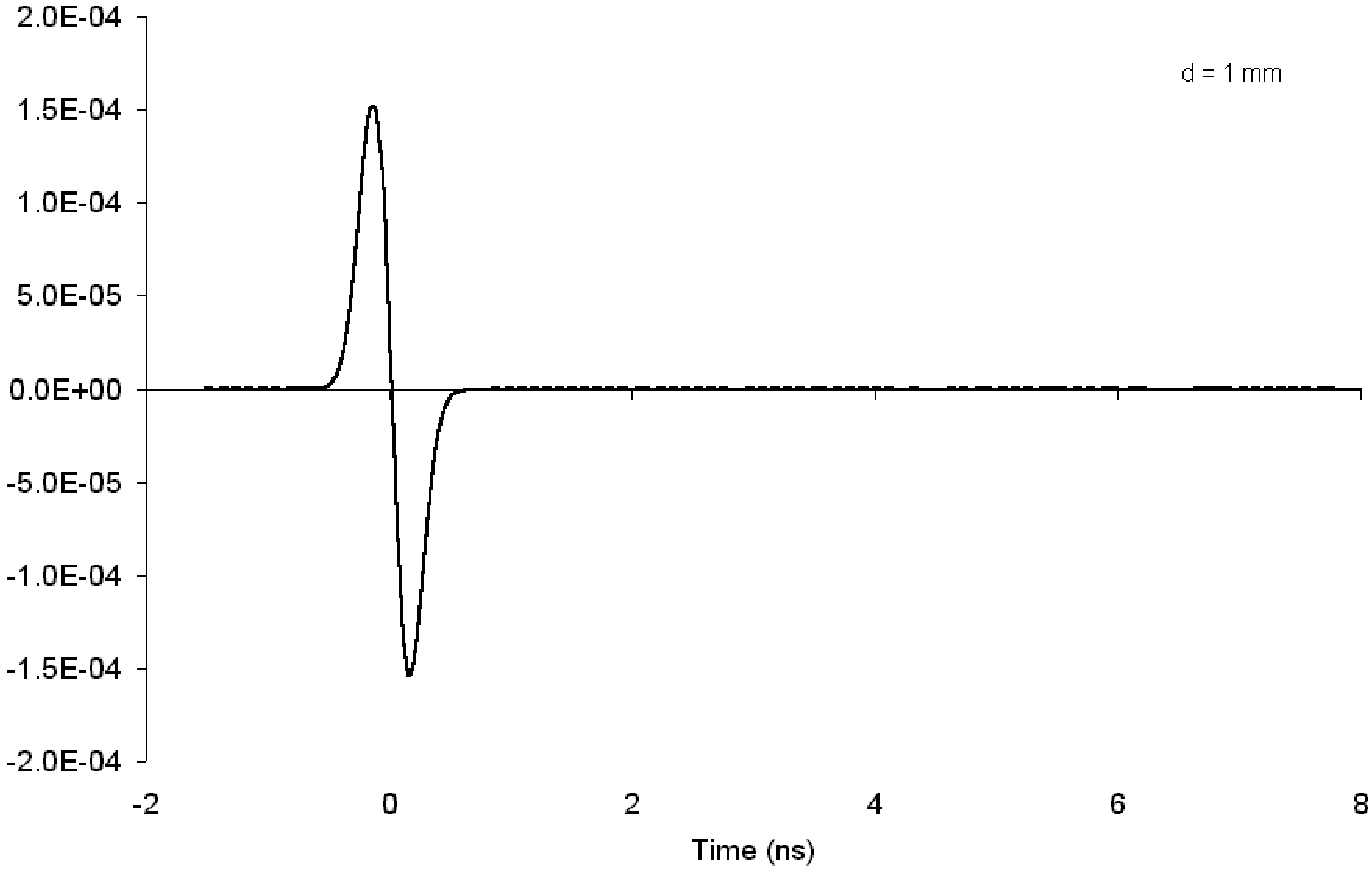
$$\boldsymbol{H} = j\omega \mathbf{s}$$

$$\mathbf{h} = \mathbf{s}\delta'(t - r/c)$$

$$y * \mathbf{h} = C_o \mathbf{s}\, \delta''(t)$$

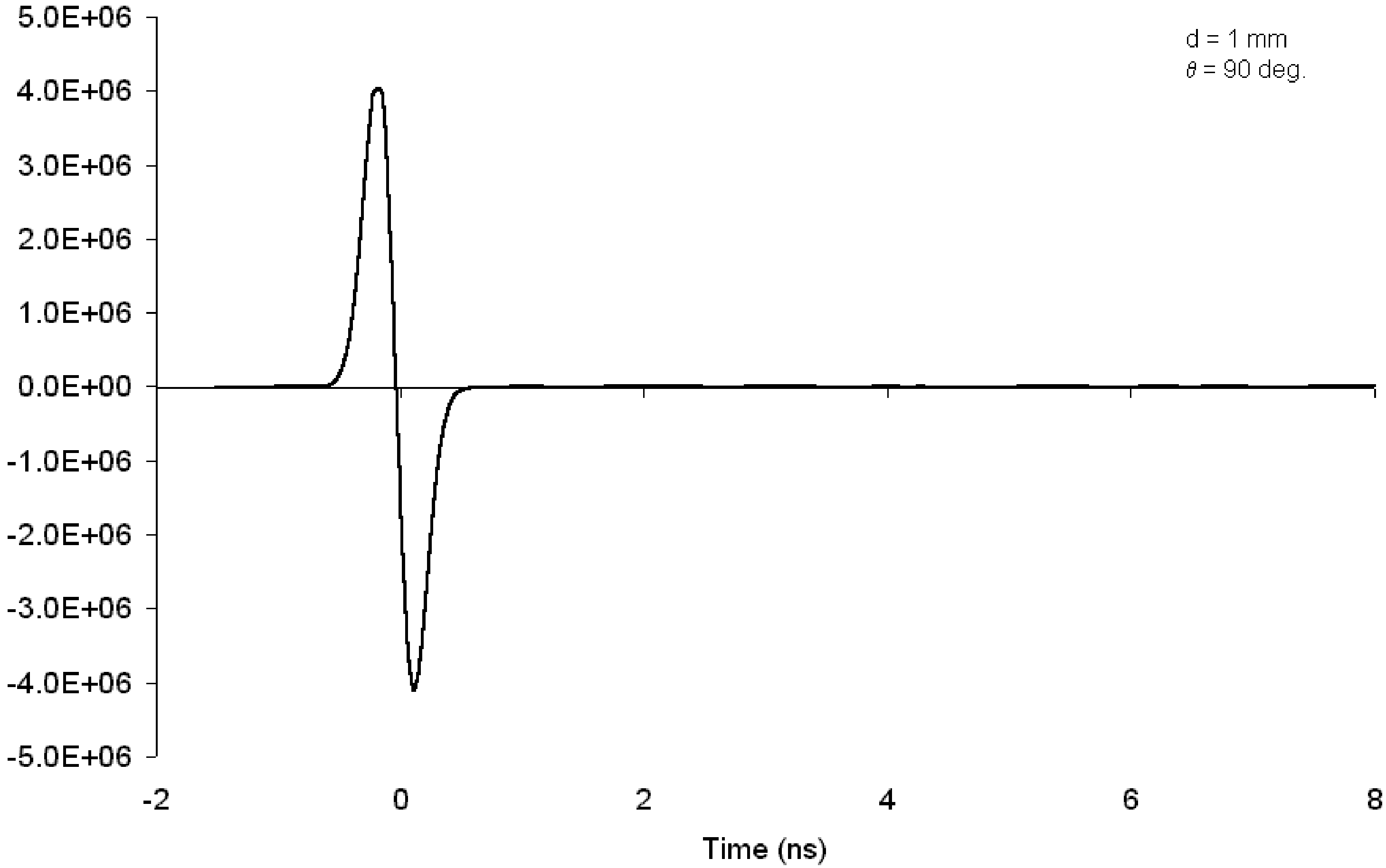
y Function of a Short Dipole

(with Gaussian source)



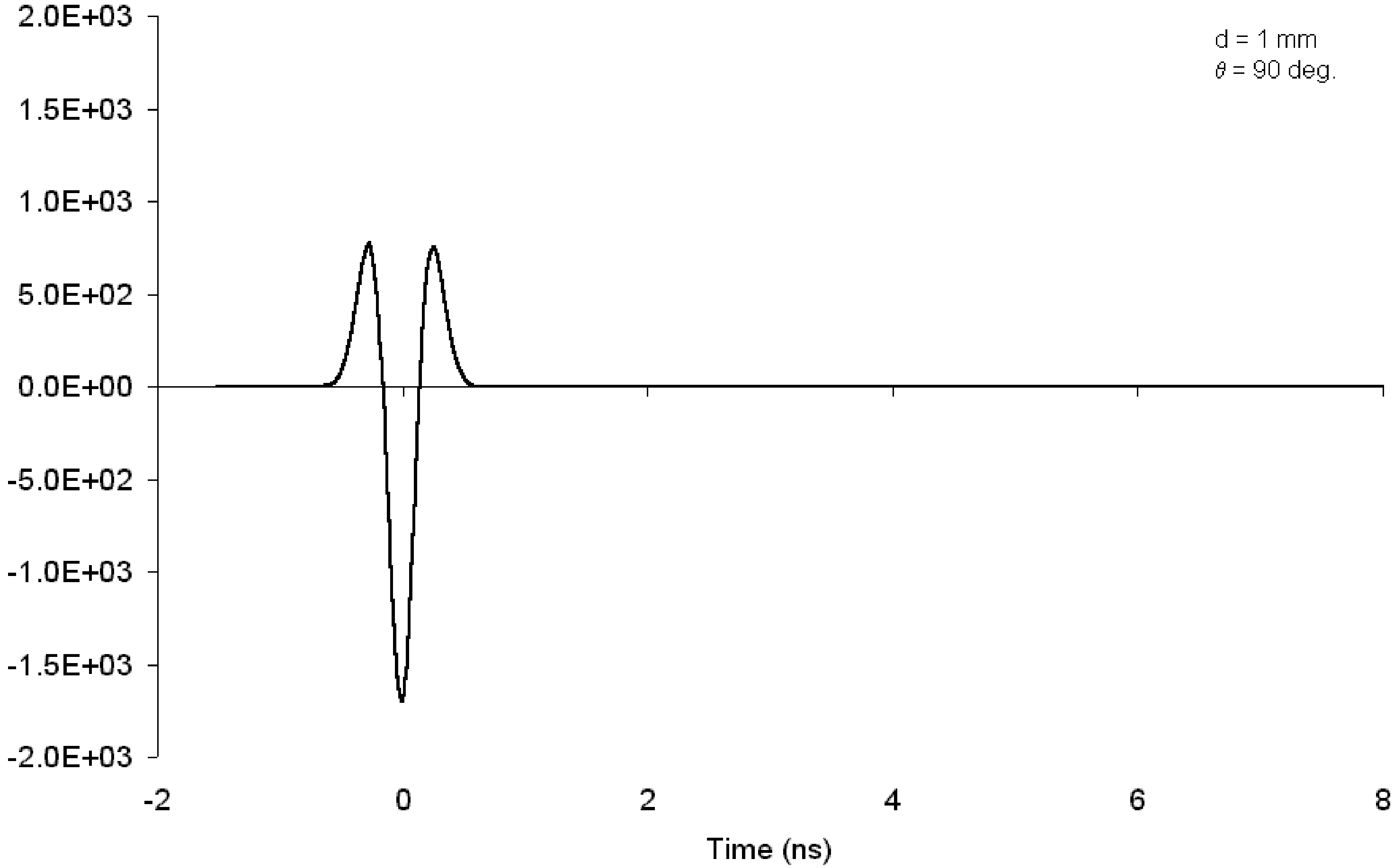
h Function of a Short Dipole

(with Gaussian source)



$y * \mathbf{h}$ of Short Dipole

(with Gaussian source)



Link Between Two Short Dipoles

Link function between two (parallel and co-planar) short dipoles

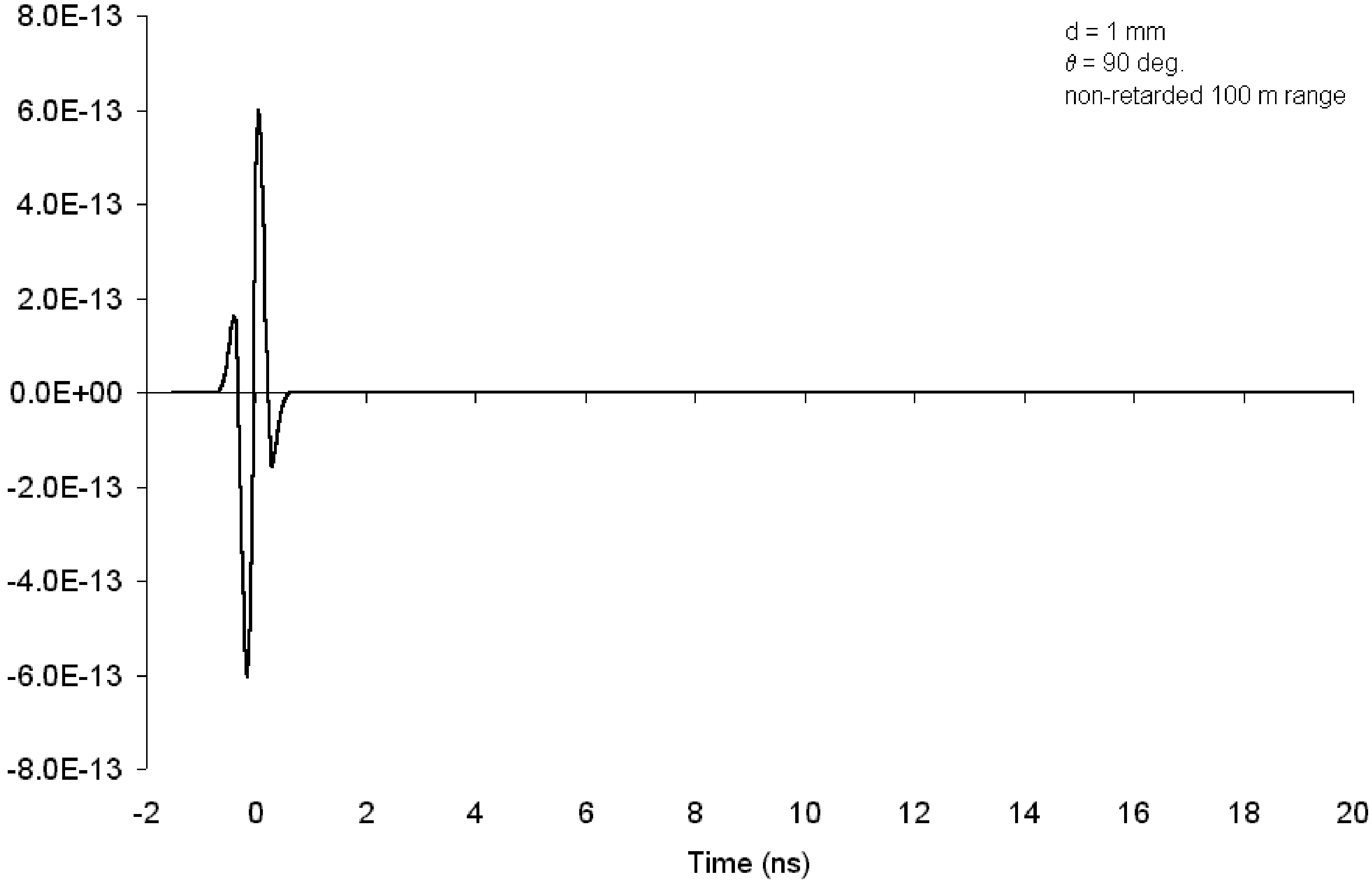
$$l = s^2 \delta''''(t - r / c)$$

Received current for unit impulse transmit voltage

$$i_r = \left(\frac{10^{-7}}{r} \right) s^2 \delta'''(t - r / c)$$

Link Between Two Short Monopoles

received current for 1 v Gaussian source



Long Thin (Straight) Dipole

"Long" means that the wavelength of highest frequency of interest is much shorter than the dipole,

"Long" means that the travel time along length is much longer than the shortest time interval of interest,

"Thin" means that the radius is very much less than the length,

Example used is length $h = 1.0$ m, radius $a = 0.1$ mm.

Electric Field of a Long Dipole

Electric field (for example 90 deg. to dipole axis),

- Requires accurate knowledge of current distributions,
- Use method-of-moments to find accurate current at all frequencies of interest and then use inverse Fourier transform.

y and **h** Functions of a Long Dipole

y function

Use results from method of moments calculations, or

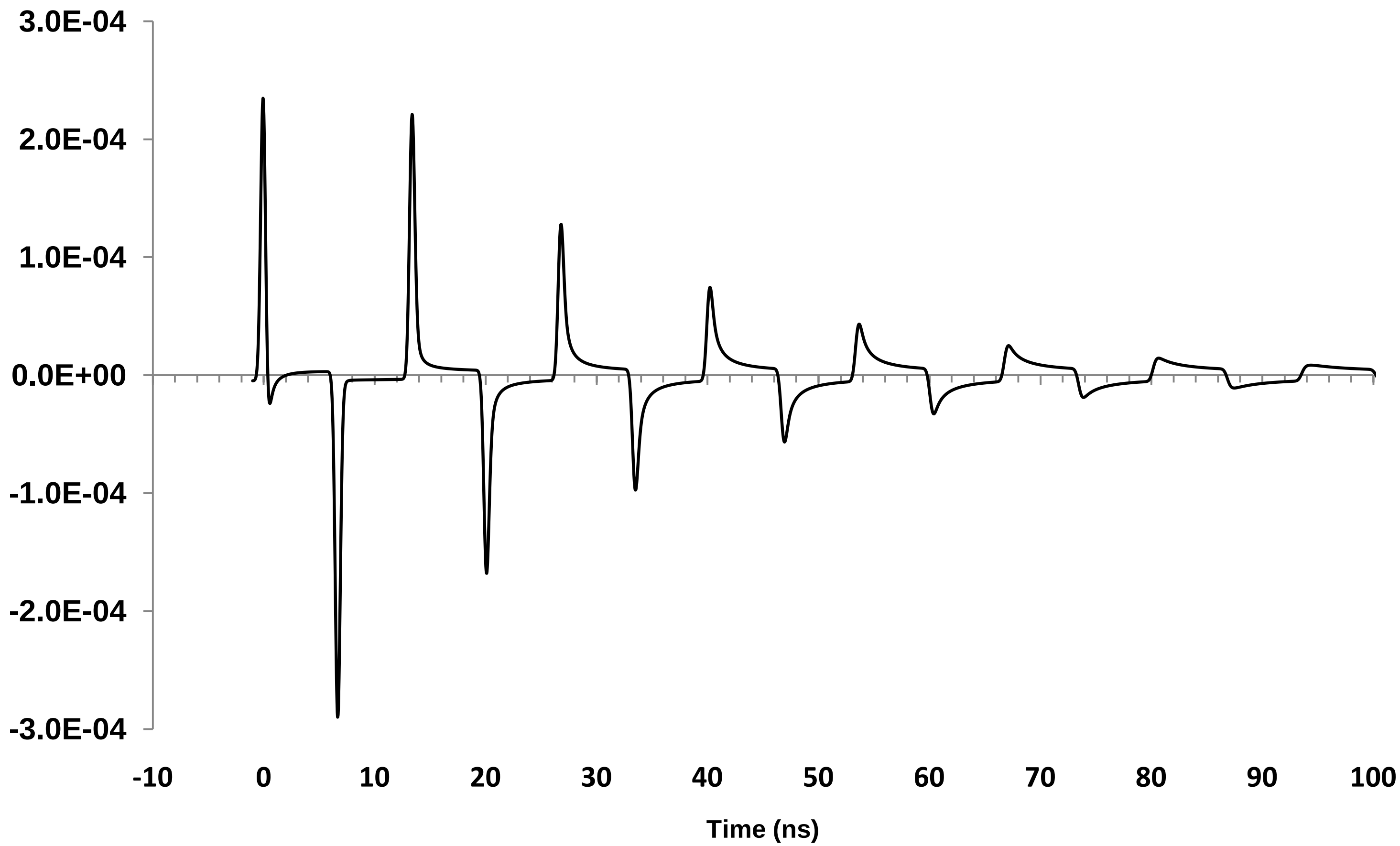
Use available analytical expressions such as those by Schelkunoff.

h function,

Inverse Fourier transform of the electric field normalized to 1 A of source current at all frequencies.

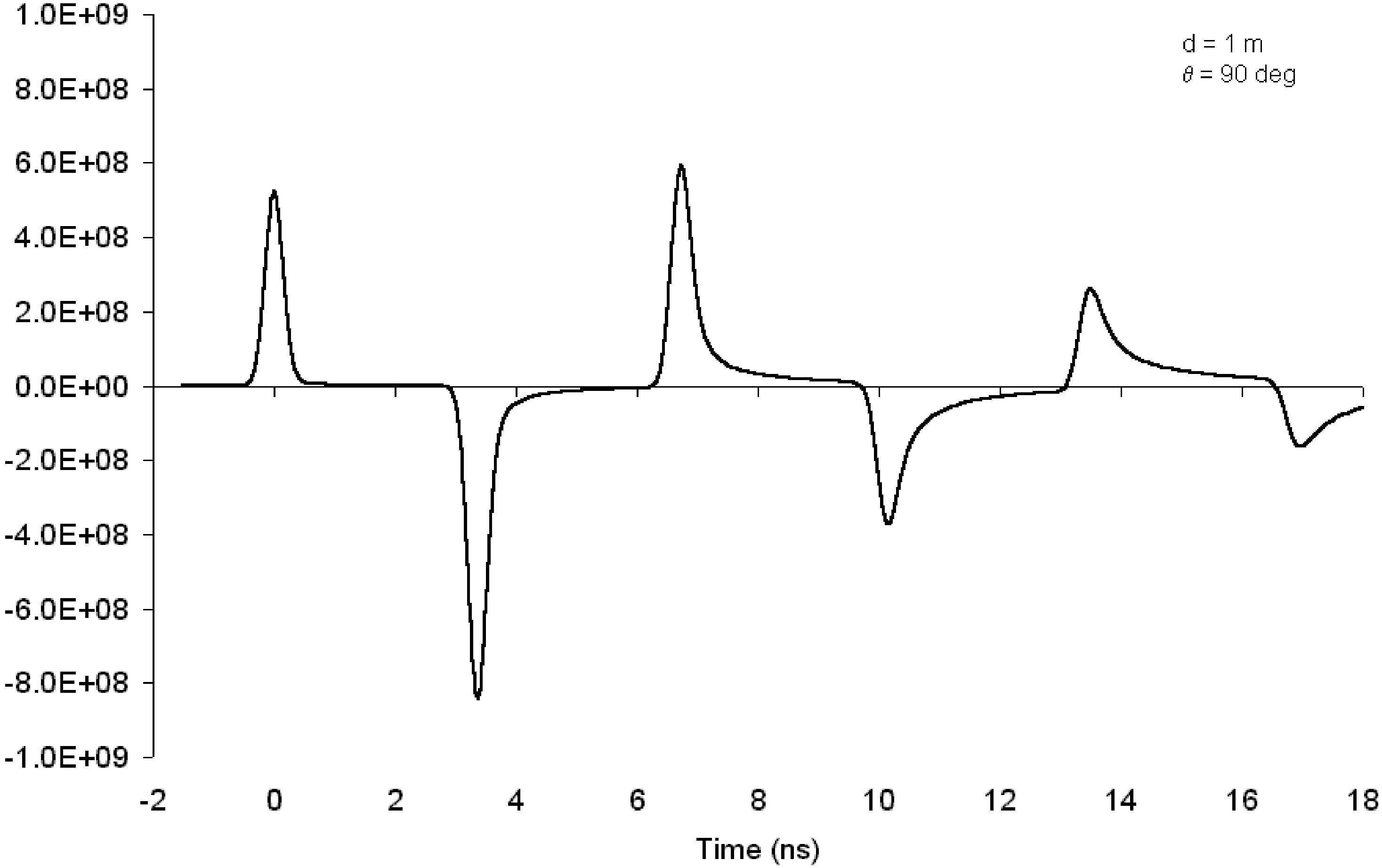
y Function of a Long Dipole

(with zero resistance Gaussian voltage source)



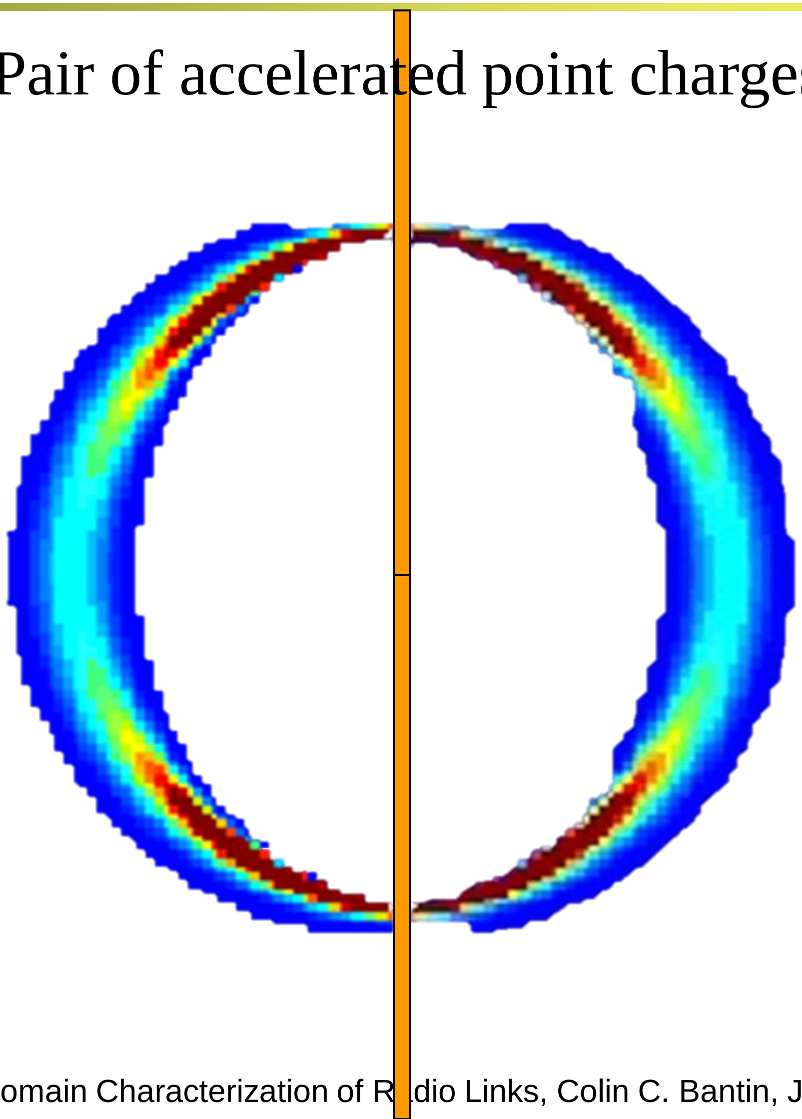
h Function of a Long Dipole

(with Gaussian source)

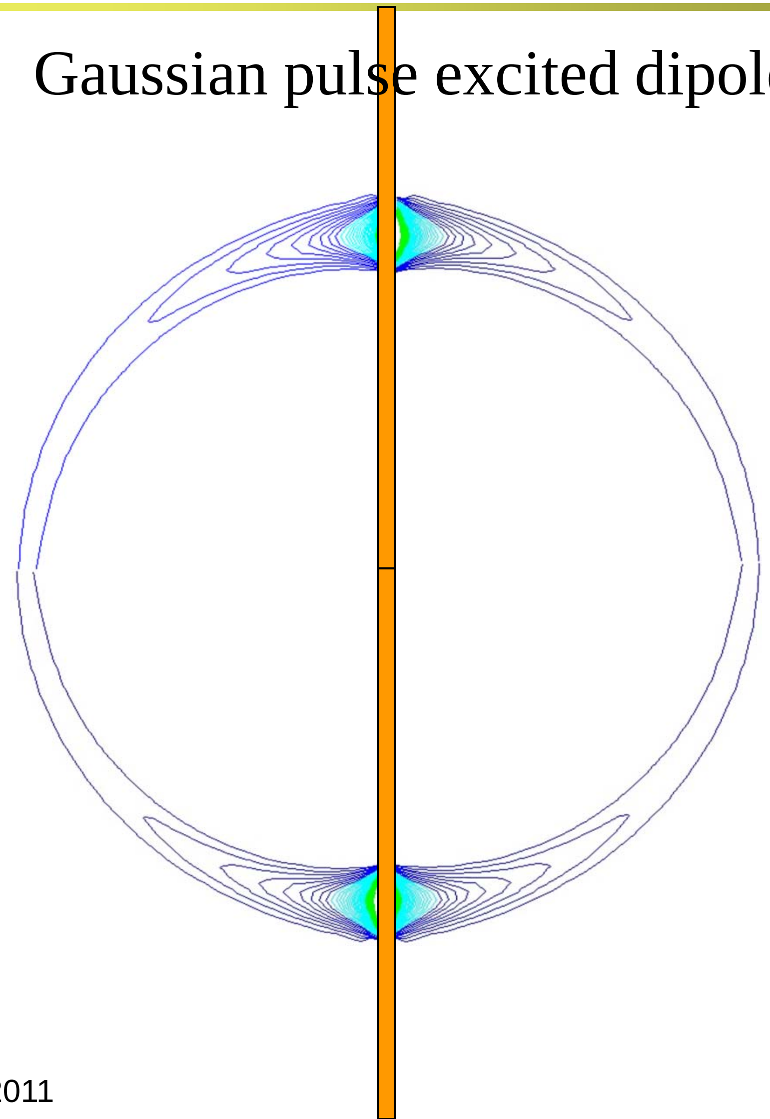


Electric Fields from Expanding Charges

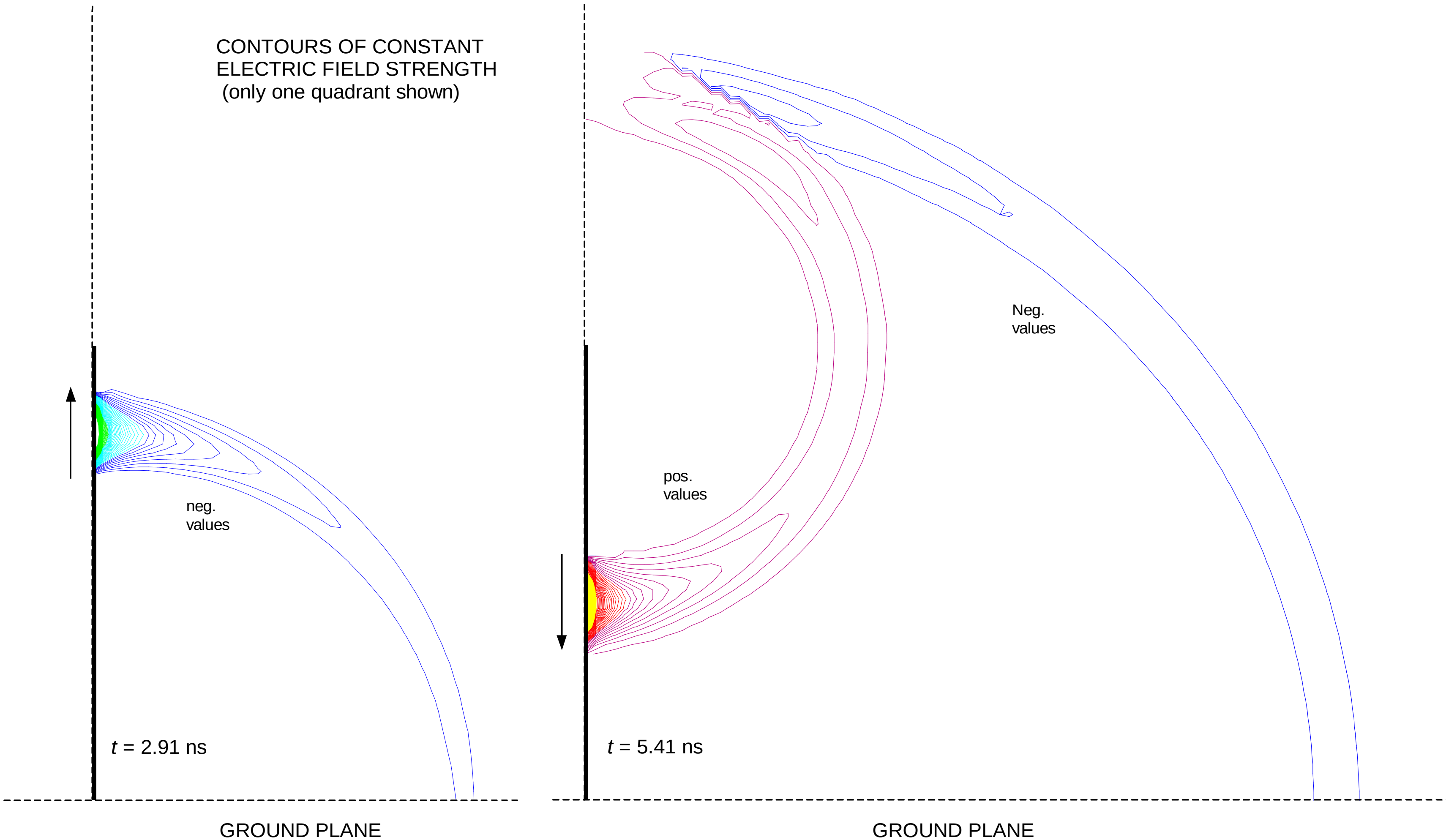
Pair of accelerated point charges



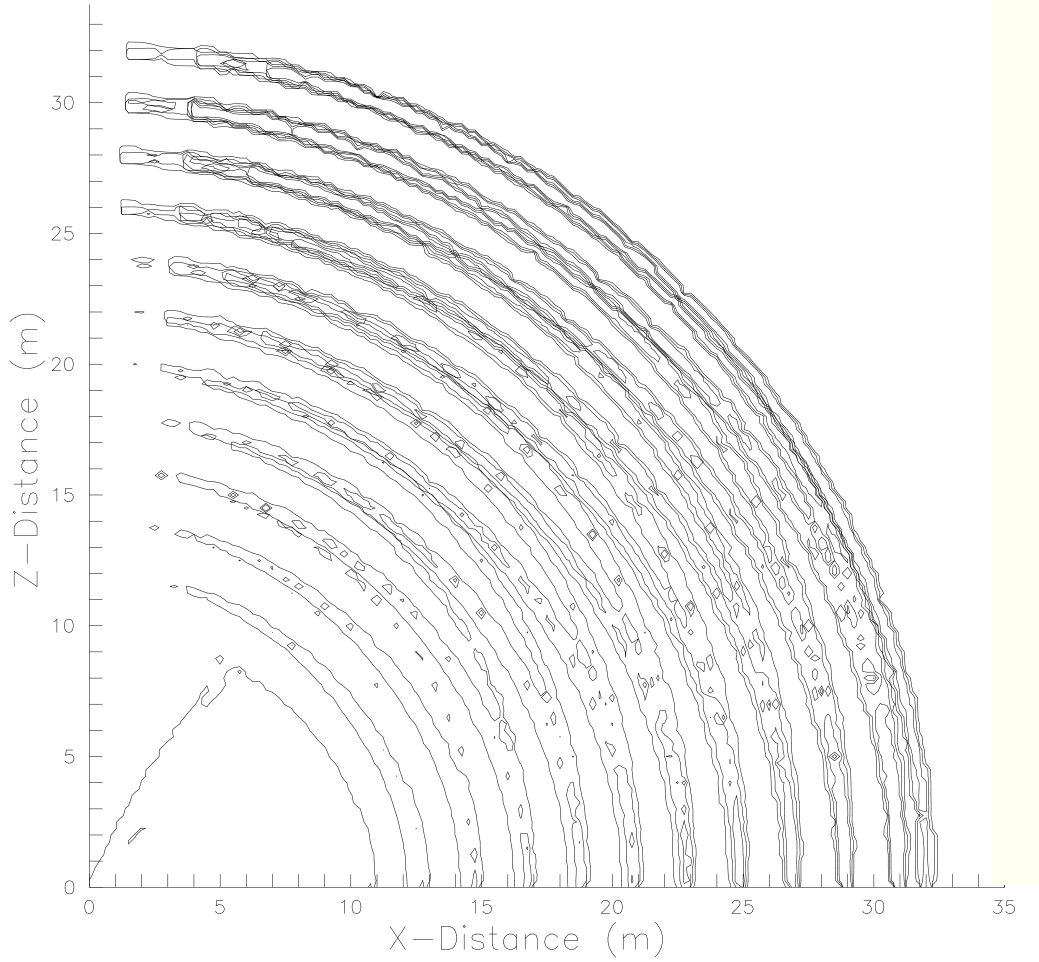
Gaussian pulse excited dipole



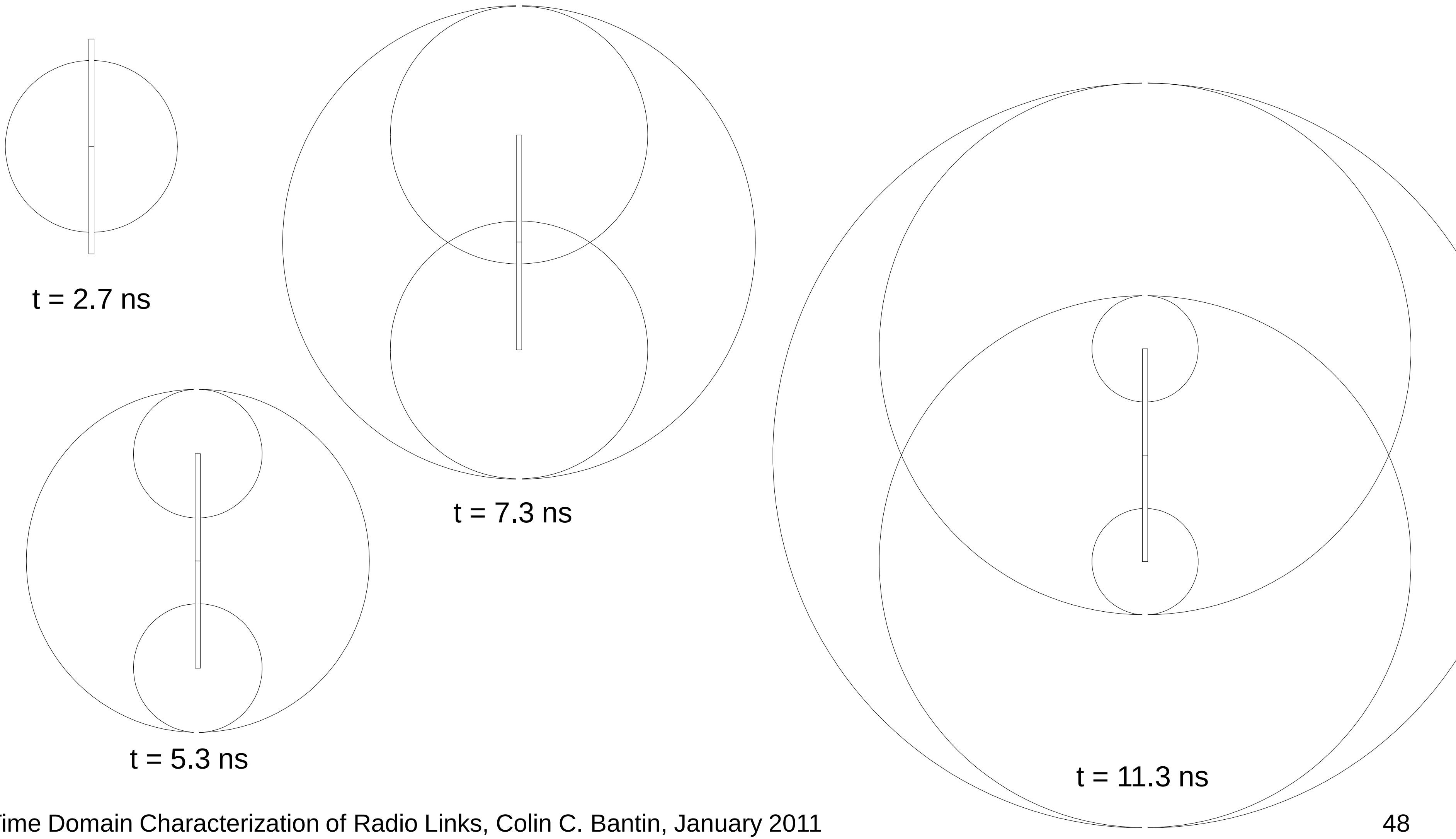
Contours of Constant Electric Field Strength



Electric Field Strength After Long Time Period



Electric Field from Short-circuit Impulse Voltage Source



Long Dipole with Reflection Matching Resistor

Arrange for reflected wave at the feed point from one tip to cancel the transmitted wave from the other tip

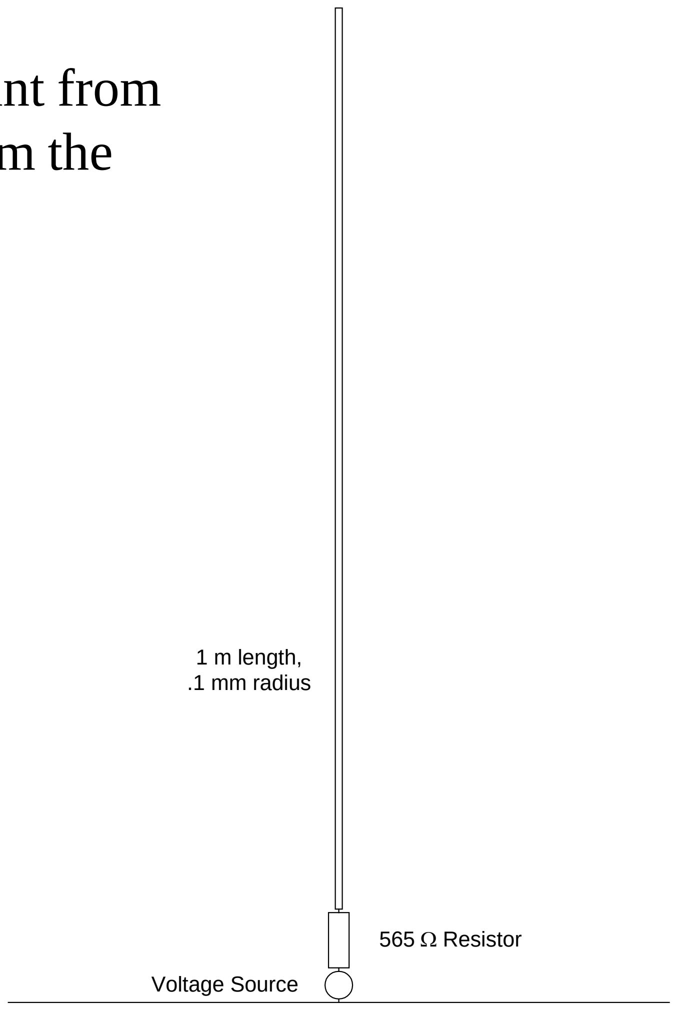
$$\mathrm{T} = -\Gamma, \quad 1-\Gamma = \Gamma, \quad \Gamma = \frac{1}{2}$$

$$\frac{R-R_0}{R+R_0} = \frac{1}{2}$$

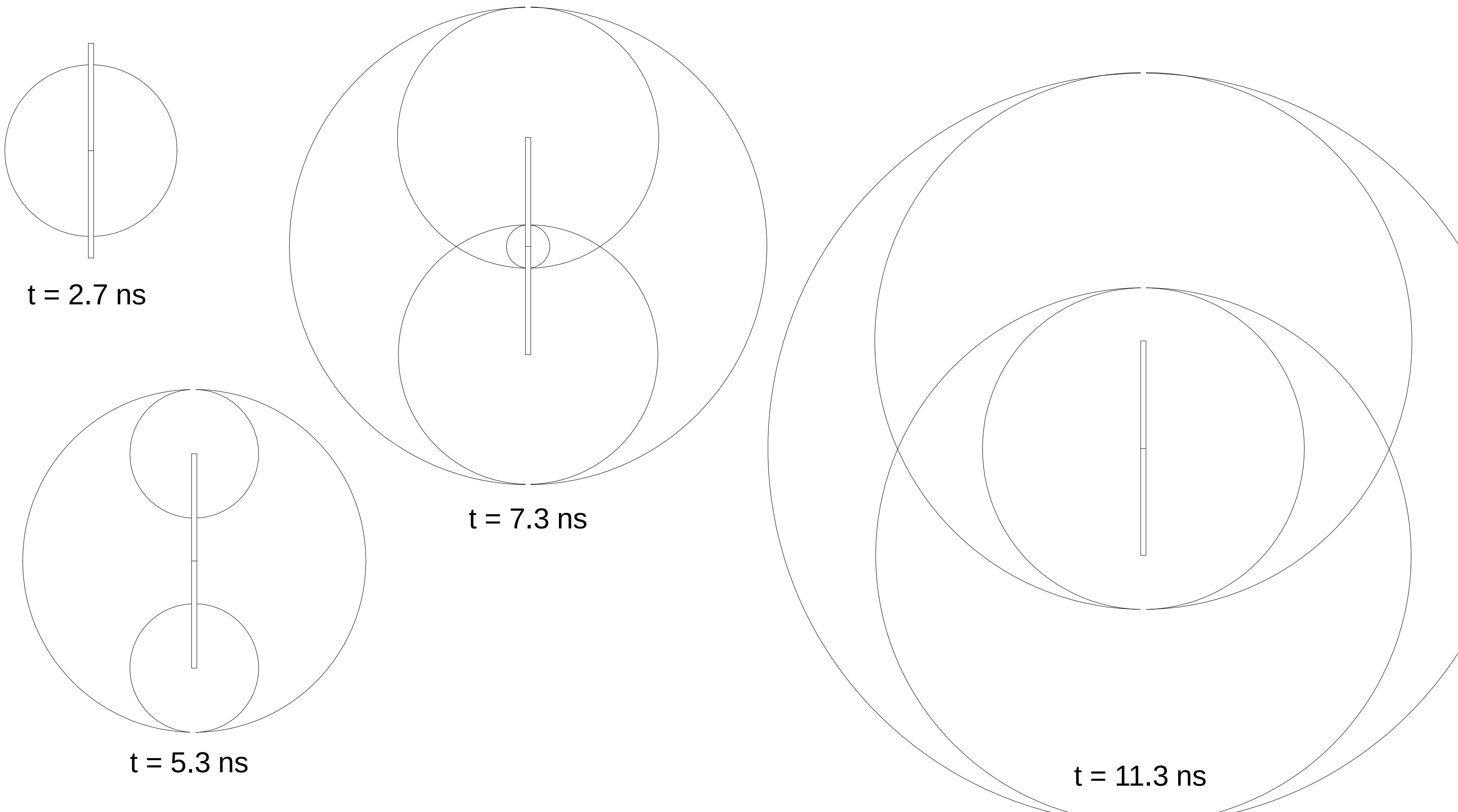
$$R = 3R_0$$

$$R_0 = \frac{\mathbf{E}}{\mathbf{H}} = 377\,\mathrm{Ohms}$$

$$R = 1131\,\mathrm{Ohms}$$

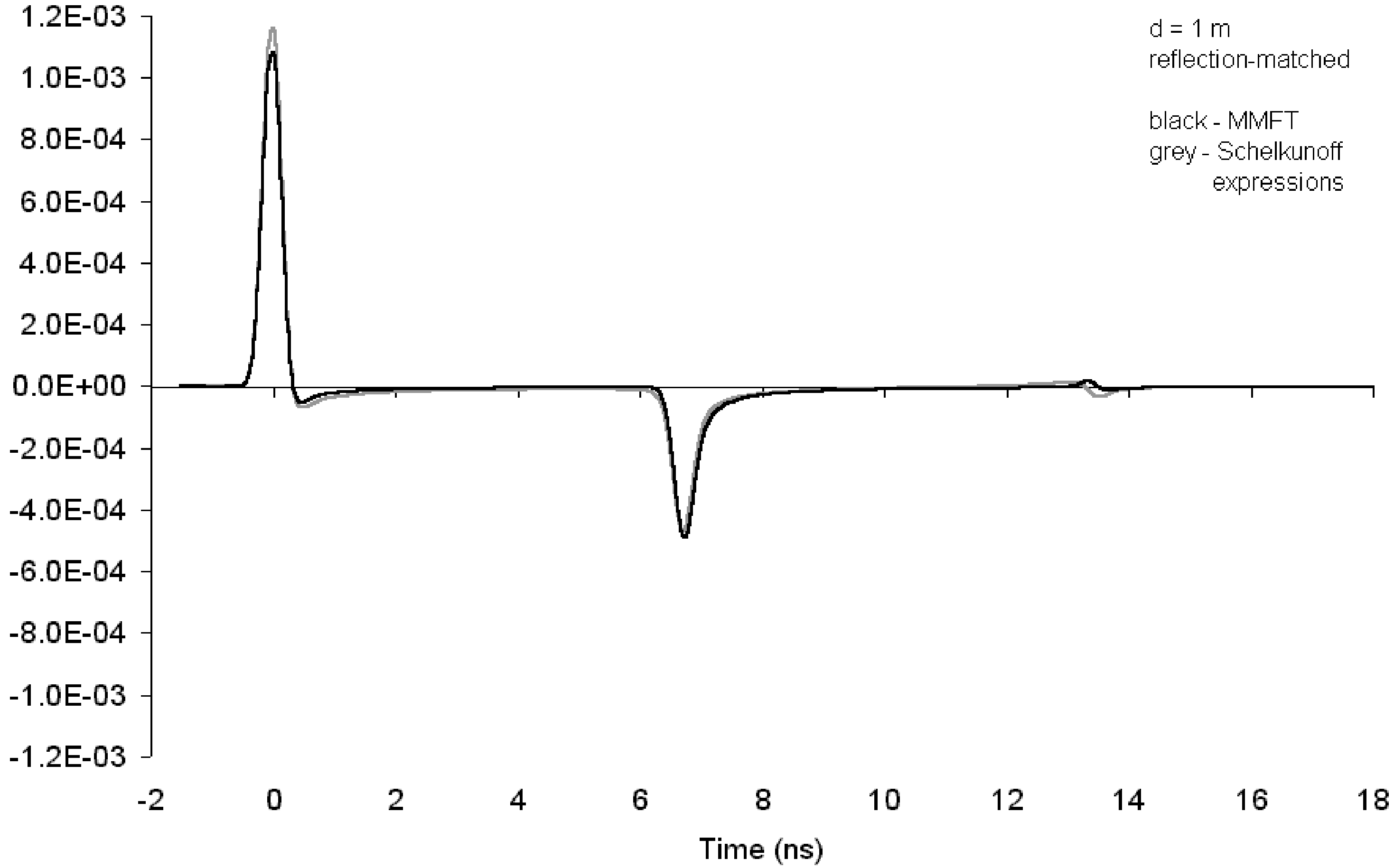


Electric Field from a Reflection-matched Impulse Voltage Source



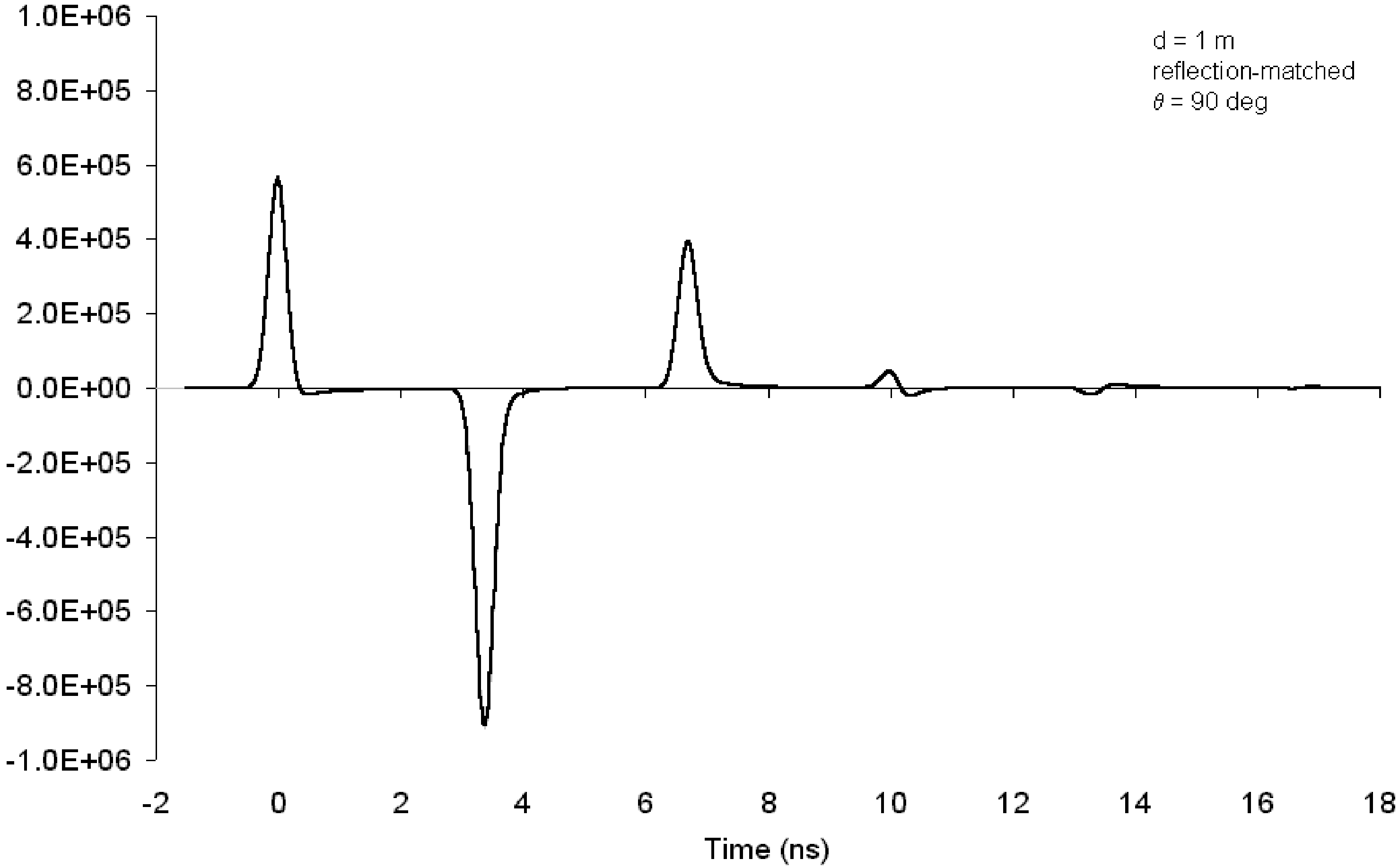
y Function of a Reflection Matched Long Dipole

(with Gaussian source)



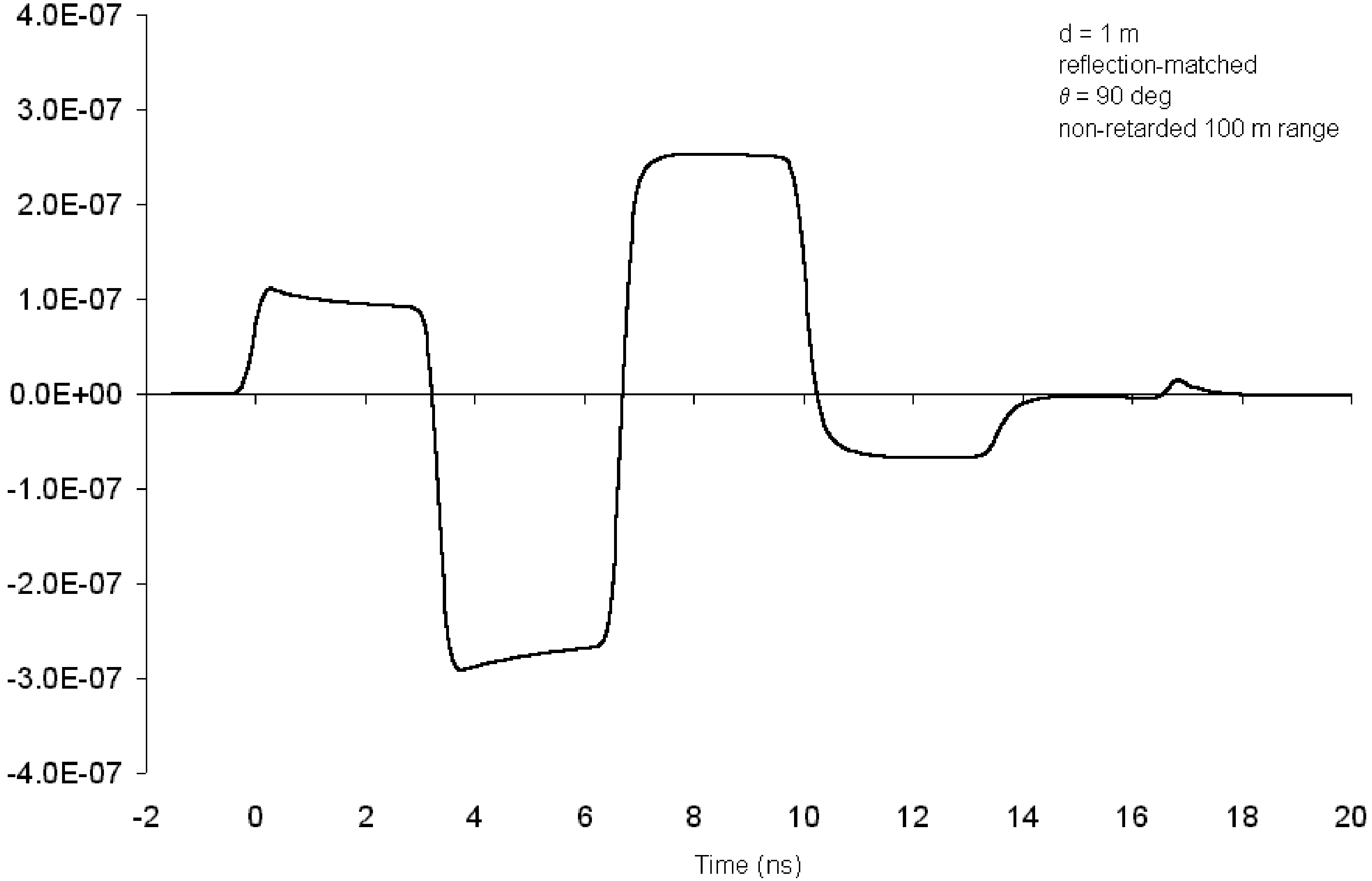
$y * \mathbf{h}$ of a Refection Matched Long Dipole

(with 1 V Gaussian source)



Link Between Two Reflection Matched Long Dipoles

(received current for 1 v Gaussian source)



Infinite Dipole

"Infinite" means that the travel time along the length is much longer than the longest observation time interval of interest,

"Infinite" means that the effects of the tip are never seen,

Examples used are;

- Radius a = 0.1 mm for cylindrical wire,
- Cone half-angle = $\tan^{-1}(10^{-4})$ for conical wire.

Electric Field of an Infinite Dipole

Electric field (for example 90 deg. to wire axis),

- Use method-of-moments results for long monopole,
- Truncate the time-domain results before tip effects are seen,
- Extrapolate smoothly (to zero) at large times.

y and **h** Functions of an Infinite Dipole

y function,

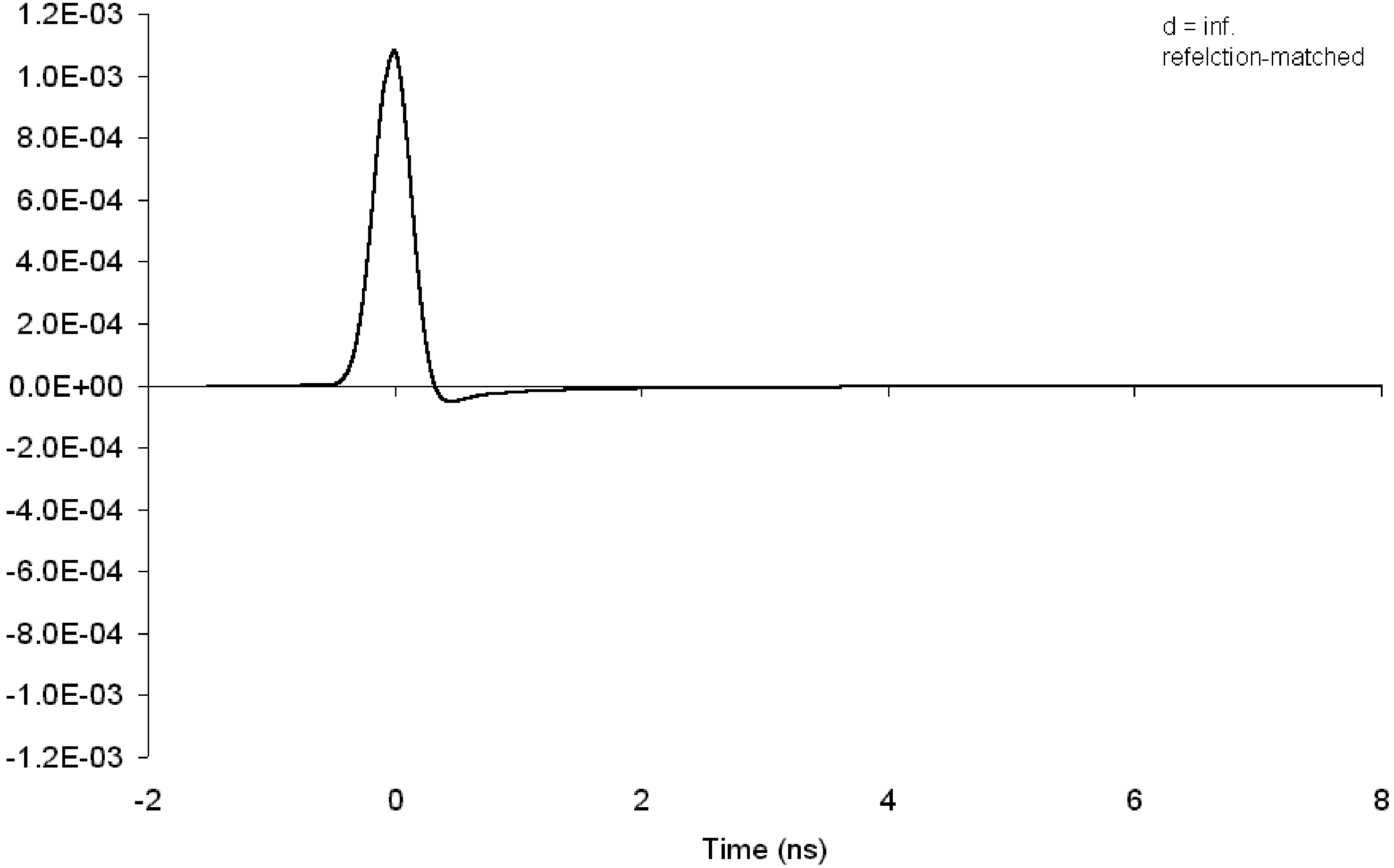
- Use results from truncated/extrapolated method of moments calculations.

h function,

- Use method-of-moments results for long monopole,
- Truncate before tip effects are seen,
- Extrapolate smoothly (to zero) at large times.

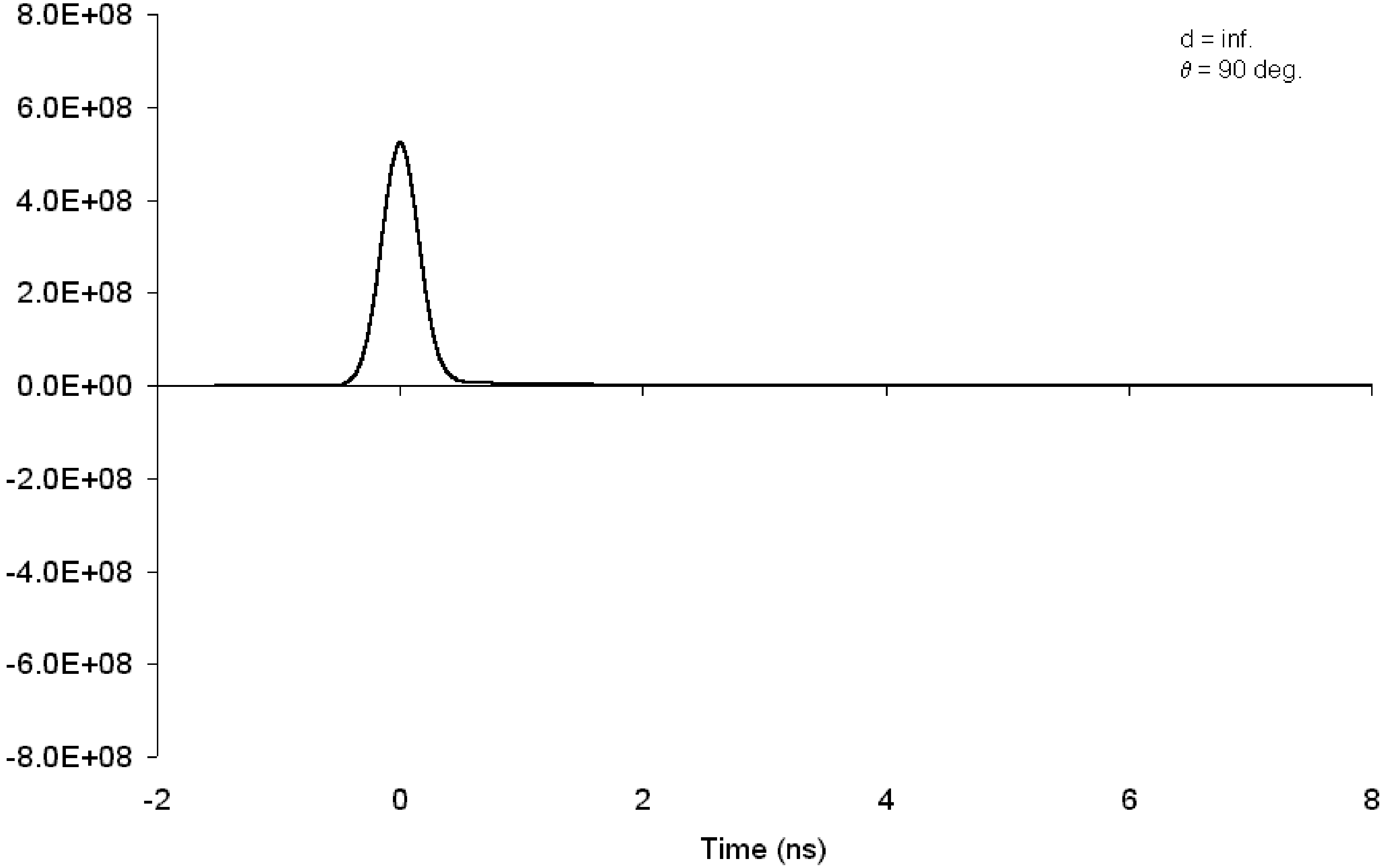
y Function of an Infinite Dipole

(with Gaussian source)



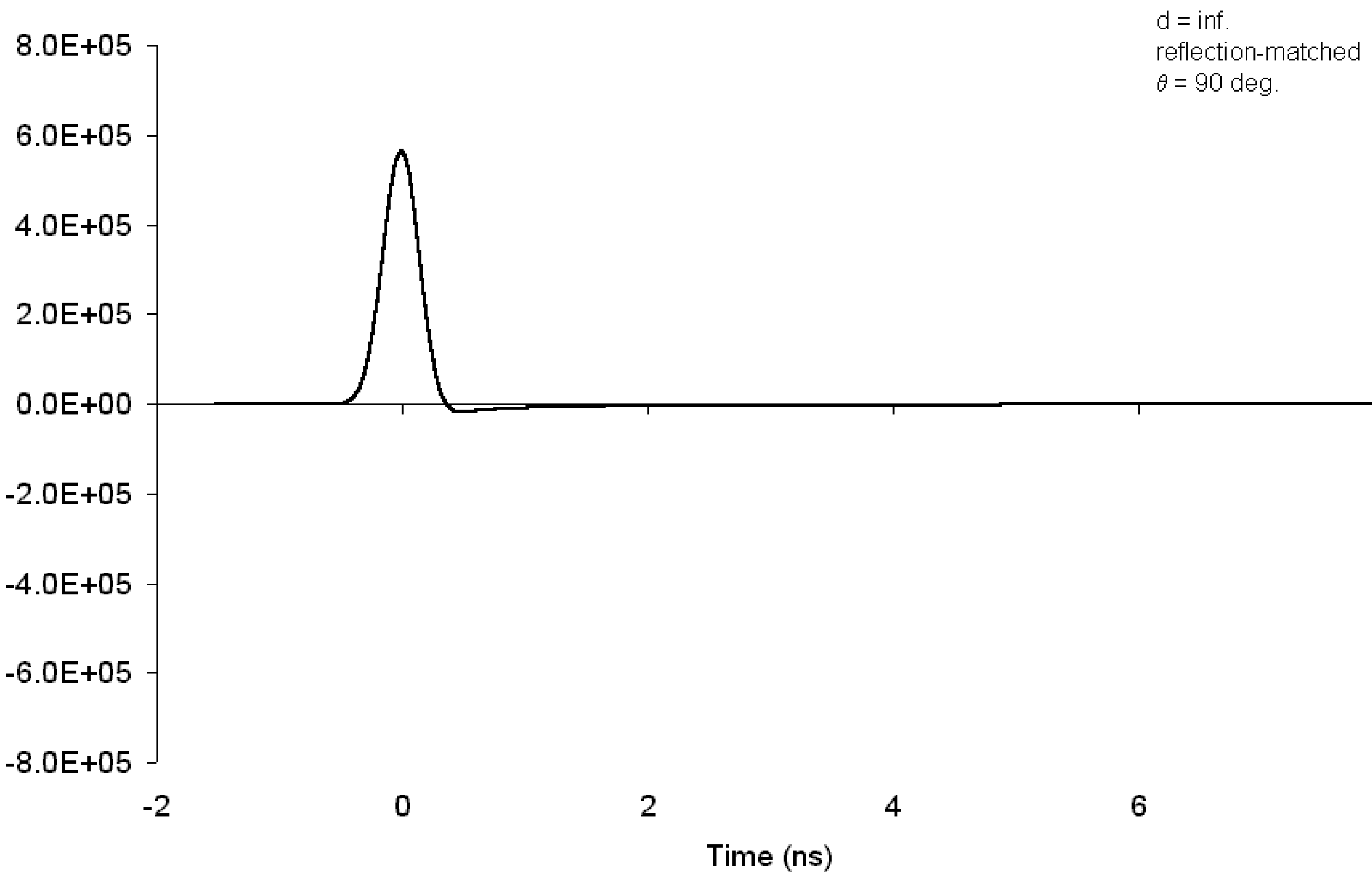
h Function of an Infinite Dipole

(with Gaussian source)



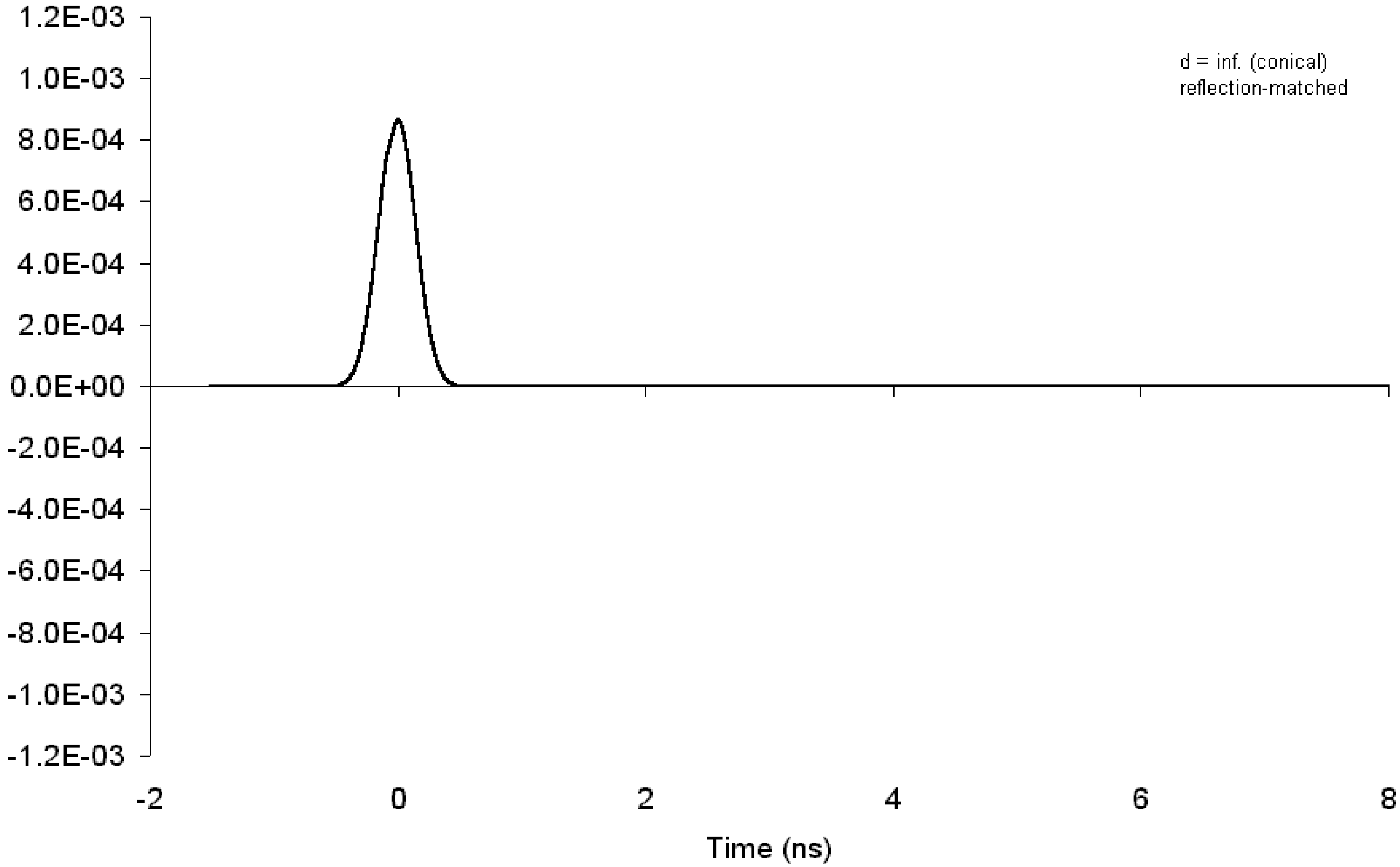
$y_* \mathbf{h}$ of an Infinite Dipole

(with Gaussian source)



y Function of an Infinite Tapered Dipole

(conical wire, with Gaussian source)



Sinusoidal Current Filament (SCF) Dipole

Same length as the long dipole,

Thin wire replaced by current filament along the axis of the wire,

Current distribution along the wire is assumed to be,

$$\mathbf{I}(z) = I_o \sin(\beta(l - z))\,\hat{\mathbf{z}}$$

for all values of β (at all frequencies).

Electric Field of a SCF Dipole

Electric field (for example 90 deg. to wire axis),

$$\mathbf{E} = \frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r} I_o \frac{2}{\beta}\frac{\left(1-\cos\left(\beta l\right)\right)}{\sin\left(\beta l\right)}\hat{\mathbf{z}}$$

y and **h** Functions of a SCF Dipole

y function,

Z usually found from the induced EMF method, hence

$$Y = 1/Z$$

$$y = \Im^{-1}\{Y\}$$

h function,

$$\boldsymbol{H} = j2c\left(\frac{1-\cos(\beta l)}{\sin(\beta l)}\right)$$

h function is not defined

Modified Impedance Function of a SCF Monopole

Write electric field as,

$$\mathbf{E} = 10^{-7}\,\frac{e^{-j\beta r}}{r}\,\frac{V_s}{Z_m}\left(j2c\left(1-\cos(\beta\, l)\right)\sin(\beta\, l)\right)$$

where

$$Z_m = Z\sin^2(\beta\, l) \qquad (Z \text{ from induced EMF method})$$

y' and **h**′ Functions of a SCF Dipole

Define a y' function

$$Y' = \frac{1}{Z_m}$$

$$y' = \Im^{-1}\{Y'\}$$

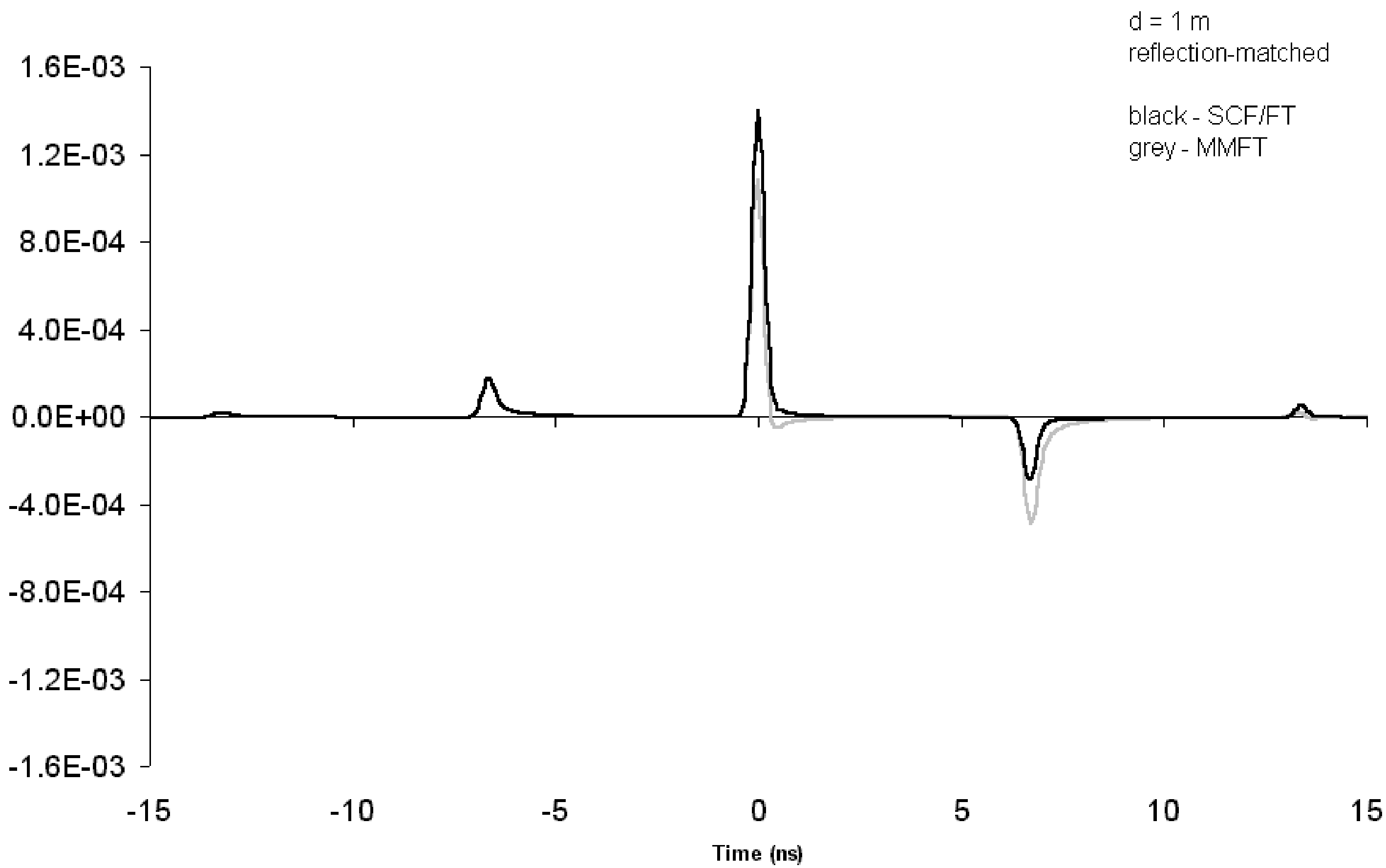
Define **h′** function as,

$$\boldsymbol{H}' = j2c(1-\cos(\beta l))\sin(\beta l)$$

$$\mathbf{h}' = c\left[-.5\delta\left(t+\frac{2l}{c}\right)+\delta\left(t+\frac{l}{c}\right)-\delta\left(t-\frac{l}{c}\right)+.5\delta\left(t-\frac{2l}{c}\right)\right]$$

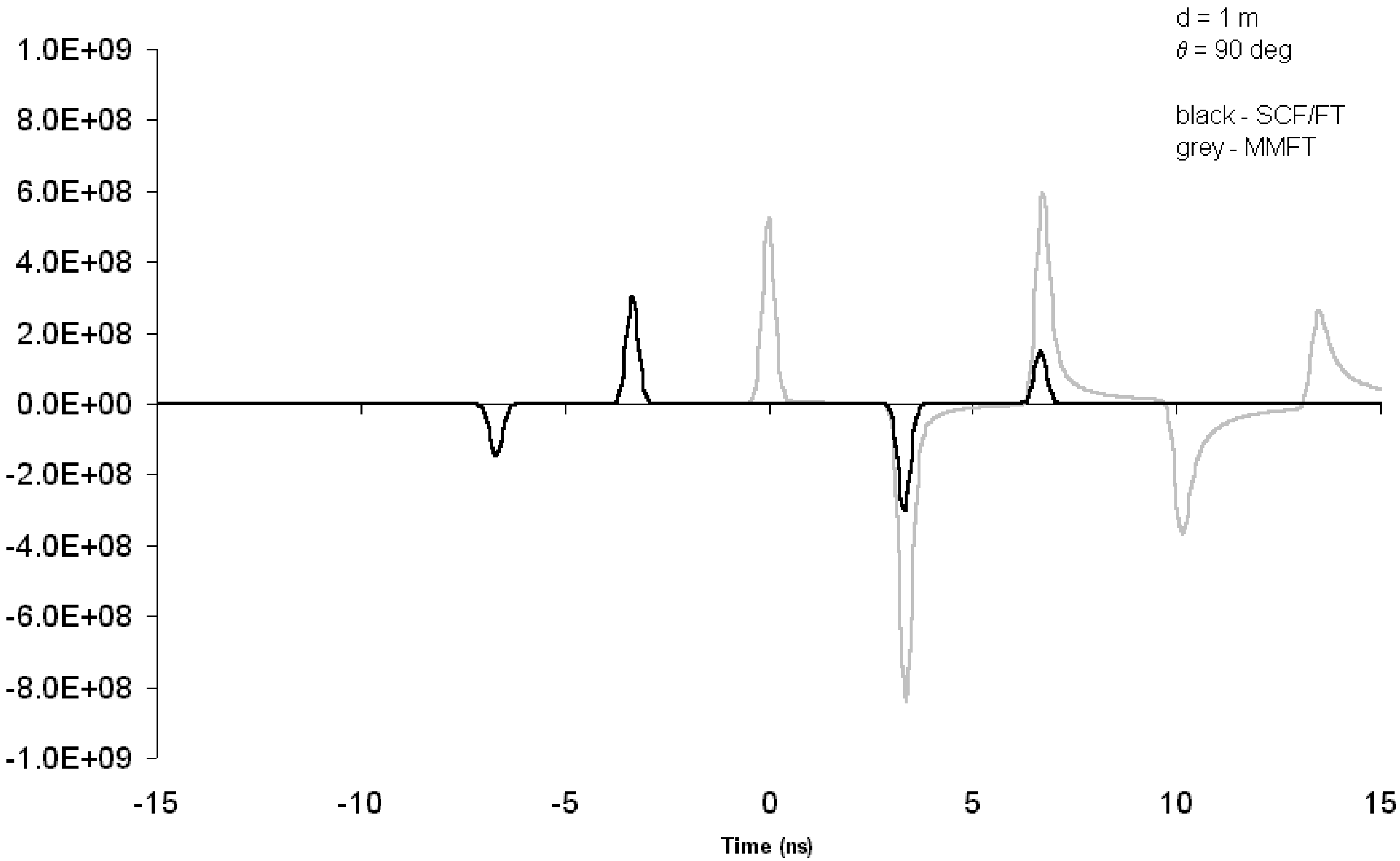
y' function of a SCF monopole

(with 1 V Gaussian source)



h′ function of a SCF monopole

(with Gaussian source)



Realizing a SCF Dipole

Food for thought:

A true SCF distribution results in a non-causal response

y' and $\mathbf{h}'$ functions have responses that occur before the source pulse is applied.

A pure SCF distribution can never be achieved in practice

The current distribution on a thin wire dipole can never approach a SCF exactly no matter how thin the wire

Summary of Characterizing Functions

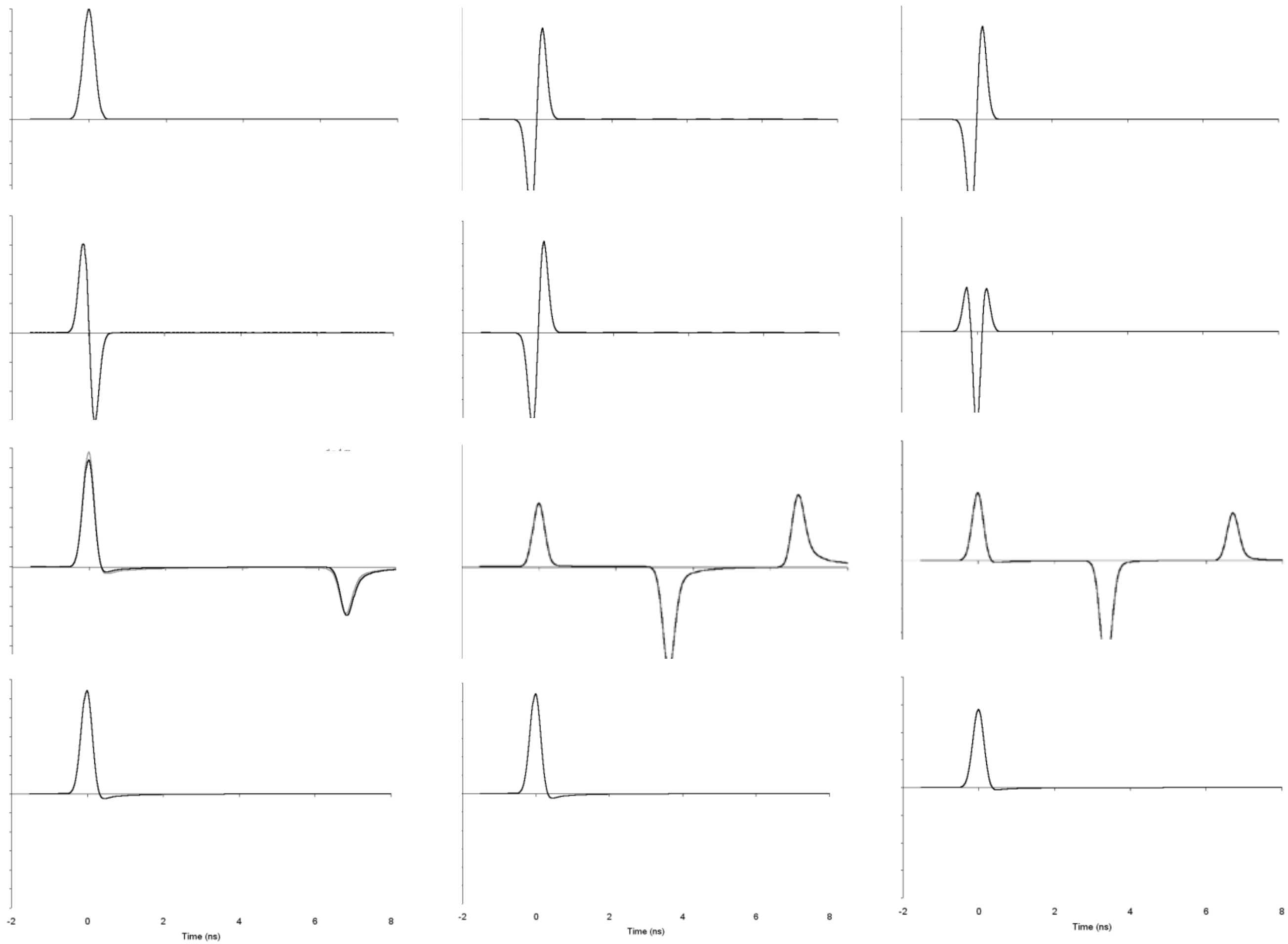
y function | $\mathbf{h}$ function | $y{*}\mathbf{h}$ function

Current Element

Short Monopole

Long Monopole

Infinite Monopole

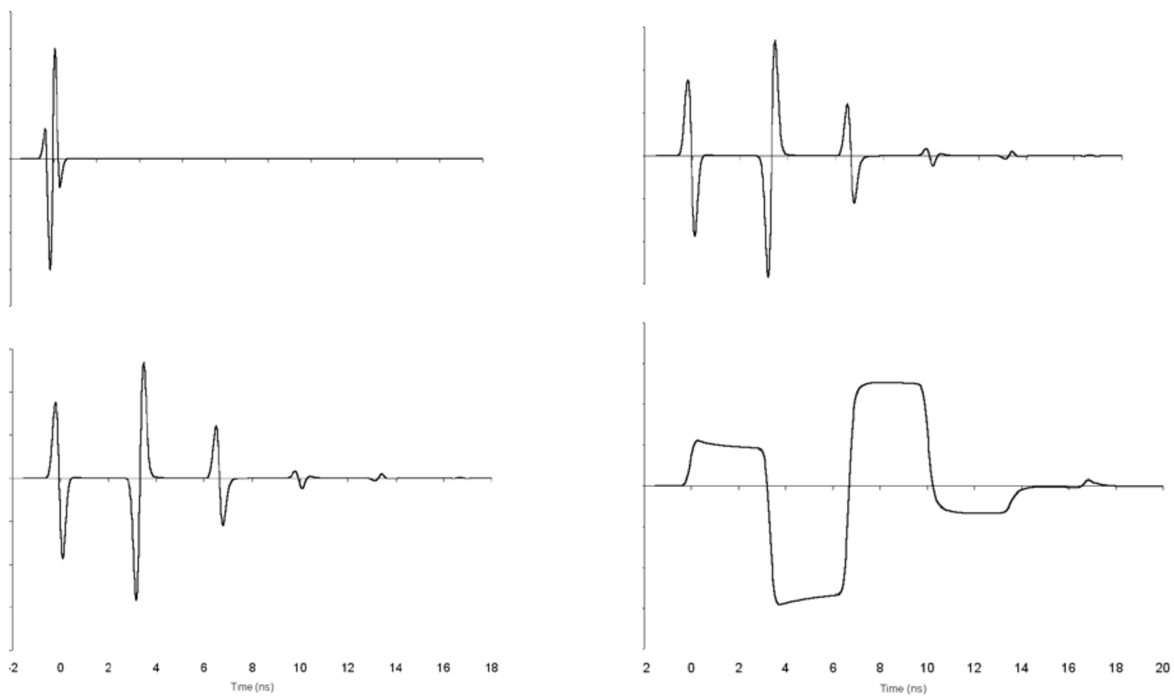


Summary of Link Functions

(non-retarded, 100 m range)

to — Short Monopole — Long Monopole

from — Short Monopole — Long Monopole



Additional Information for Real World Links

Link calculations developed so far do not include a realistic propagation media,

- y and $\mathbf{h}$ functions were derived for free space conditions,
- No account taken for absorption, refraction, defraction or reflections from objects in the transmission path.

Need to add the propagation characteristics of the particular link,

- Medium is often air but can include propagation through walls,
- Reflections from wall and buildings in very common,
- Multiple reflections from tunnels is of particular interest.

Introduce a propagation factor, $p(t)$, into the link equations to account for frequency (therefore time) dependent propagation factors,

$$i_r = \frac{10^{-7}}{r} \int y_t * y_r * \mathbf{h}_t \otimes \mathbf{h}_r * p * v_s \, dt$$

Determining Propagation Factors

- Simple cases can be done analytically,
  - Perfectly flat ground with known ε and σ characteristics,
  - Single, isolated reflecting surfaces with known ε and σ characteristics,
- More complex cases can be treated numerically,
  - Ray tracing programs to determine the often large number of multiple reflecting paths between the transmitter and the receiver,
- Complex cases can also be treated by measurements,
  - Swept frequency measurements to determine the link values, and de-convolution process to isolate the propagation environment from the antenna characteristics,
  - Separate measurements to determine the antenna characteristics.
- Numerical analysis and a measurement program are the subjects of ongoing research,
  - Profs. Serras and Hum and their students ay the U of T, in conjunction with Thales Rail Signalling Solutions Inc., with a focus on urban rail tunnels

Conclusions

Antennas can be characterized in the time domain by;

- A *y* function that represents the "admissivity" of the antenna as seen by the source,
- An **h** function that represents the integrated current density distribution on the antenna structure as seen at some angle relative to the antenna orientation.

Links between antennas can be characterized in the time domain by;

- A time integration of the convolution of the *y* and **h** functions of the two antennas and a voltage source function, and a propagation function (evaluated at the appropriate retarded time), and a scale factor of $10^{-7} / r$,
- Real life cases can be examined through numerical modelling and measurement.

Significant insights can be gained into the radiation process by employing a time domain analysis and using thin wire dipoles as examples;

- Specific origin of radiation,
- Control of tip reflections from a long dipole,
- Illustrates the time integration process involved in receiving an electric field.

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